

# PAIRS AND GROUPS OF GENETICALLY RELATED LONG-PERIOD COMETS AND PROBABLE IDENTITY OF THE MYSTERIOUS LICK OBJECT OF 1921

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## ABSTRACT

We present the history of investigation of the dynamical properties of pairs and groups of genetically related long-period comets (other than the Kreutz sungrazing system). Members of a comet pair or group move in nearly identical orbits and their origin as fragments of a common parent comet is unquestionable. The only variable is the time of perihelion passage, which differs from member to member considerably due primarily to an orbital-momentum increment acquired during breakup. Meter-per-second separation velocities account for gaps of years or tens of years, thanks to the orbital periods of many millennia. The physical properties of individual members may not at all be alike, as illustrated by the trio of C/1988 A1, C/1996 Q1, and C/2015 F3. We exploit orbital similarity to examine whether the celebrated and as yet unidentified object, discovered from the Lick Observatory near the Sun at sunset on 1921 August 7, happened to be a member of such a pair and to track down the long-period comet to which it could be genetically related. Our search shows that the Lick object, which could not be a Kreutz sungrazer, was most probably a companion to comet C/1847 C1 (Hind), whose perihelion distance was  $\sim 9 R_{\odot}$  and true orbital period approximately 8300 yr. The gap of 74.4 yr between their perihelion times is consistent with a separation velocity of  $\sim 1 \text{ m s}^{-1}$  that set the fragments apart following the parent's breakup in a general proximity of perihelion during the previous return to the Sun in the 7th millennium BCE.

*Subject headings:* comets: general — methods: data analysis

## 1. INTRODUCTION

Unverified sightings of bright objects near the Sun, reported from time to time, are usually explained as incidental observations of known cosmic bodies, such as the planets Venus, Jupiter, or Mercury, when stationary over a period of seconds or minutes; or of various man-made items in the atmosphere, such as weather or other balloons, when seen to float.

It happens much less often that an unidentified object near the Sun is reported by an expert. The proper procedure is to promptly alert the scientific community, so that the body could be detected by other observers, its existence confirmed, and its nature established. If the object orbits the Sun, then depending on the quality of gathered observations, which in each case rests on the conditions and circumstances at the times of discovery and follow-up observations, information may sometimes be gained to secure the motion of the object accurately enough to eventually allow an orbit to be computed.

If the observations do not allow an orbit determination, the lost object almost always remains unidentified. Rare exceptions are lucky situations, when the object happens to be a long-period comet (defined in this study by an orbital period between  $\sim 100$  and  $\sim 100\,000$  yr; Section 4.3) that is genetically related to another, previously observed comet and is, with reasonable confidence, recognized as such. As fragments of a common parent, the two make up a *comet pair*. If there are more than two fragments, they all belong to the same *comet group* or *family*. Since the members of a comet pair or group move about the

Sun in virtually identical orbits, the motion of any one of them can readily be *emulated* to a fairly high degree of accuracy by equating it with the motion of any other one, the perihelion time being the only variable.

A well-known case in point is a comet discovered in the Sun's outer corona during the total solar eclipse on 1882 May 17. Scientific expeditions, dispatched to Sohag, on the Nile in Upper Egypt, to observe the eclipse, reported visual and photographic detections of a bright streak not aligned with the radial coronal ejections but nearly tangential to the occulted limb — the comet's narrow tail (e.g., Abney & Schuster 1884). The official designation of the comet is now X/1882 K1, but informally it has been known as *Tewfik*, named in honor of Tewfik Pasha, then reigning Khedive of Egypt. Observed for only 72 seconds (the duration of totality at Sohag; e.g., Tacchini 1883, Abney & Schuster 1884), the comet has never again been seen since. It turned out that the astrometric position of the sharply defined nucleus of X/1882 K1, published by Abney & Schuster (1884), was within  $\sim 0'.5$  of the orbit of comet C/1843 D1 ((Marsden 1967), a major member of the Kreutz sungrazer system; and that the position of X/1882 K1 reported by Tacchini (1882, 1883)<sup>1</sup> was similarly close to the orbit of C/1880 C1 (Marsden 1989a), another Kreutz member, which reached perihelion only 2.3 years before the eclipse. This evidence is the basis for a general consensus that X/1882 K1, too, was a member of the Kreutz system.

<sup>1</sup> Kreutz (1901) mistakenly attributed Tacchini's (1882, 1883) position of X/1882 K1 to C. Trépied, and this error seems to have been propagating in the literature throughout the 20th century. The idea that the prominent feature was a comet did not occur to Trépied (1882) until after he saw Schuster's photographs.

Table 1

ORBITAL ELEMENTS OF COMETS C/1988 F1 (LEVY) AND C/1988 J1 (SHOEMAKER-HOLT) (EQUINOX J2000.0).

Orbital element/Quantity	Comet C/1988 F1	Comet C/1988 J1
Osculation epoch (TT)	1987 Nov. 21.0	1988 Feb. 9.0
Time of perihelion passage $t_\pi$ (TT)	1987 Nov. 29.94718	1988 Feb. 14.22162
Argument of perihelion $\omega$	326°.51491	326°.51498
Longitude of ascending node $\Omega$	288°.76505	288°.76487
Orbit inclination $i$	62°.80744	62°.80658
Perihelion distance $q$	1.1741762	1.1744657
Orbital eccentricity $e$	0.9978157	0.9978301
Orbital period $P$ (yr) $\left\{ \begin{array}{l} \text{osculation} \\ \text{original}^a \end{array} \right.$	$12\,460 \pm 460$ 13 960	$12\,590 \pm 230$ 13 840
Orbital arc covered by observations	1988 Mar. 22–1988 July 18	1988 May 13–1988 Oct. 20
Number of observations employed	30	60
Root-mean-squares residual	$\pm 0''.7$	$\pm 0''.9$
Orbit-quality code <sup>b</sup>	2A	2A
Reference	Marsden (1989b)	Marsden (1989b)

<sup>a</sup> With respect to the barycenter of the Solar System.<sup>b</sup> Following the classification system introduced by Marsden et al. (1978); the errors of the elements other than the orbital period are unavailable.

## 2. PAIRS AND GROUPS OF GENETICALLY RELATED LONG-PERIOD COMETS

Because of implications for the evolution of comets and a relevance to the problem of fragmentation, comet pairs and groups have been of scientific interest for a long time. While the genetic makeup of the Kreutz sungrazing system is certain (Section 1), lists of pairs and groups of other comets, based on apparent orbital similarity, were published, for example, by Pickering (1911) (49 pairs and 17 larger groups) and by Porter (1952, 1963) (more than a dozen groups). The issue of genetically associated comets got to the forefront of scientific debate in the 1970s, when a major statistical investigation by Öpik (1971) led to his startling conclusion that at least 60% of all comets with aphelia beyond  $\sim 10$  AU were members of one of a large number of comet groups. In response, Whipple (1977), combining a Monte Carlo approach with probability methods, countered that a random sample of comets has clumping properties very similar to those of Öpik’s set of observed comets and that there is no compelling evidence for any overabundance of genetically related comets. Supporting Whipple’s conclusion, Kresák (1982) found no true comet pairs at all.

### 2.1. The First Pair

By coincidence, strong evidence for the existence of pairs of genetically related long-period comets — other than the Kreutz sungrazers — became available several years after the debate ended. The circumstances clearly favored Whipple’s (1977) conservative approach.

The first comet pair consisted of C/1988 F1 (Levy) and C/1988 J1 (Shoemaker-Holt). They were discovered within two months of each other and their perihelion times were only 76 days apart. Their osculating orbits, computed by Marsden (1989b) and listed in Table 1, were virtually identical, yet their true orbital period, defined by the original barycentric semimajor axis, was close to 14 000 yr. The striking orbital similarity and a very small temporal separation both suggested that the two comets still had been part of a single body during the previous return to perihelion around 12 000 BCE — and for a long time afterwards.

On the other hand, the relative motions of C/1988 F1 and C/1988 J1 are inconsistent with those of fragments of an ordinary split comet except when observed many years after separation. Survival of secondary fragments (or companions) over such long periods of time is untypical — weeks or months are the rule — among the split short-period comets, the most notable exceptions being 3D/Biela (Marsden & Sekanina 1971; Sekanina 1977, 1982) and 73P/Schwassmann-Wachmann (Sekanina 2006).<sup>2</sup> And the gap of 76 days in the perihelion time, minuscule in terms of the orbital period of C/1988 F1, is unusually protracted when compared to an average split comet, whose fragments are commonly separated by a fraction of a day or a few days at the most.

The motion of a companion relative to the primary is known to be affected by both an orbital-momentum change at breakup (resulting in a nonzero separation velocity) and a continuous outgassing-driven differential nongravitational acceleration after separation (Sekanina 1978, 1982). As the dominant radial component of the nongravitational acceleration causes the less massive — and usually the intrinsically fainter — companion to start trailing the more massive primary in the orbit, the brighter leading fragment represents a signature of the role of the nongravitational forces at work.<sup>3</sup> We now examine the two mechanisms as potential triggers of the 76 day gap between C/1988 F1 and C/1988 J1.

#### 2.1.1. Effects of a separation velocity

Well-determined separation velocities of split comets’ fragments are known to range from  $\sim 0.1$  to  $\sim 2$  m s<sup>−1</sup> (e.g., Sekanina 1982). Keeping this in mind, we employ a simple model to demonstrate that the fragmentation event that led to the birth of C/1988 F1 and C/1988 J1

<sup>2</sup> A record of sorts might be held by the short-period comets 42/Neujmin and 53P/Van Biesbroeck, which according to Carusi et al.’s (1985) orbital integrations had had nearly identical orbits before 1850, implying that they are fragments of a single comet that split at that time near Jupiter. However, the proposed genetic relationship of the two comets appears to be a matter of ongoing discussion (e.g., Pittichová et al. 2003).

<sup>3</sup> There are rare exceptions to this rule, but even then the more massive fragment is intrinsically fainter for only a limited time.

could not have occurred during the previous return to perihelion. We begin by combining the virial theorem with Kepler's third law,

$$V_{\text{frg}} = c_0 \sqrt{\frac{2}{r_{\text{frg}}} - \frac{1}{a}}, \quad (1)$$

$$P = c a^{\frac{3}{2}},$$

where  $r_{\text{frg}}$  and  $V_{\text{frg}}$  are, respectively, the heliocentric distance and the orbital velocity at the time of fragmentation,  $a$  is the semimajor axis of the orbit,  $P$  is the orbital period, and  $c_0$  and  $c$  are constants. Expressing  $V_{\text{frg}}$  in  $\text{m s}^{-1}$ ,  $r_{\text{frg}}$  and  $a$  in AU, and  $P$  in yr, the values of the constants are  $c_0 = 2.978 \times 10^4$  and  $c = 1$ . By differentiating both equations, requiring that  $r_{\text{frg}} \ll 2a$ , and writing an increment  $\Delta(1/a)$  in terms of an increment  $\Delta P$  of the orbital period and  $1/a$  in terms of  $P$ , we obtain the following expression for the relationship between  $\Delta V$ , a separation velocity (in  $\text{m s}^{-1}$ ) in the direction of the orbital-velocity vector, and  $\Delta P$ :

$$\Delta V = \frac{c_0 c^{\frac{2}{3}} \sqrt{2}}{6} r_{\text{frg}}^{\frac{1}{2}} P^{-\frac{5}{3}} \Delta P = C r_{\text{frg}}^{\frac{1}{2}} P^{-\frac{5}{3}} \Delta P, \quad (2)$$

where  $C$  is a constant whose value is 7019 when  $\Delta P$  is in yr or 19.22 when in days. For the pair of C/1988 F1 and C/1988 J1, we find from Table 1  $P \simeq 14\,000$  yr and  $\Delta P = 76.3$  days, so that the inferred separation velocity (in  $\text{m s}^{-1}$ ) in the direction of the orbital-velocity vector becomes

$$\Delta V = 0.00018 r_{\text{frg}}^{\frac{1}{2}}. \quad (3)$$

Since  $a_{\text{orig}} \simeq 580$  AU,  $r_{\text{frg}}$  is to be *much* smaller than 1160 AU and the separation velocity at the time of fragmentation *near the previous perihelion* is predicted to be  $\Delta V < 0.006 \text{ m s}^{-1}$ , orders of magnitude lower than the least separation velocities for the split comets. Only if the separation velocity should be *exactly* normal to the orbital-velocity vector, rather an absurd premise, might this condition be satisfied. A much more plausible scenario is C/1988 F1 and C/1988 J1 having separated from their common parent fairly recently, when it already was on its way from aphelion to the 1987 perihelion.

This argument is fully supported by a more rigorous treatment of the effects on the companion's perihelion time that are caused by its separation velocity (reckoned relative to the primary) acquired upon the parent comet's fragmentation. Let the primary's perihelion time be  $t_\pi$ , its perihelion distance  $q$ , its eccentricity  $e < 1$ , and its parameter  $p = q(1+e)$ . Furthermore, let  $\mathbf{P}$ ,  $\mathbf{Q}$ ,  $\mathbf{R}$  be the unit orbit-orientation vectors with the components  $P_x, \dots, R_z$  in the right-handed ecliptical coordinate system. We wish to determine the temporal gap between the perihelion passages of the companion and the primary,

$$\Delta t_\pi = t'_\pi - t_\pi, \quad (4)$$

where  $t'_\pi$  is the companion's perihelion time; a positive  $\Delta t_\pi$  signifies that the companion trails the primary.

With the planetary perturbations and nongravitational forces ignored, the conditions at the time of fragmentation,  $t_{\text{frg}}$ , are described as follows: the comet's heliocentric distance is  $r_{\text{frg}}$  and the position vector's coordinates in the ecliptical system are  $(x_{\text{frg}}, y_{\text{frg}}, z_{\text{frg}})$ ; the primary

fragment's true anomaly is  $v_{\text{frg}} = \arccos[(p/r_{\text{frg}} - 1)/e]$ , its orbital velocity is  $V_{\text{frg}}$ , and the components of its velocity vector in the ecliptical system are  $(\dot{x}_{\text{frg}}, \dot{y}_{\text{frg}}, \dot{z}_{\text{frg}})$ ; and the companion's separation velocity relative to the primary is  $U$ , its components in the cardinal directions of the **RTN** coordinate system,<sup>4</sup>  $U_R$ ,  $U_T$ , and  $U_N$ , being related to the ecliptical components  $U_x$ ,  $U_y$ , and  $U_z$  by

$$\begin{bmatrix} U_x \\ U_y \\ U_z \end{bmatrix} = \begin{bmatrix} P_x & Q_x & R_x \\ P_y & Q_y & R_y \\ P_z & Q_z & R_z \end{bmatrix} \cdot \begin{bmatrix} \cos v_{\text{frg}} & -\sin v_{\text{frg}} & 0 \\ \sin v_{\text{frg}} & \cos v_{\text{frg}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} U_R \\ U_T \\ U_N \end{bmatrix}. \quad (5)$$

The ecliptical components of the companion's orbital velocity vector at time  $t_{\text{frg}}$  are

$$\begin{bmatrix} \dot{x}'_{\text{frg}} \\ \dot{y}'_{\text{frg}} \\ \dot{z}'_{\text{frg}} \end{bmatrix} = \begin{bmatrix} \dot{x}_{\text{frg}} \\ \dot{y}_{\text{frg}} \\ \dot{z}_{\text{frg}} \end{bmatrix} + \begin{bmatrix} U_x \\ U_y \\ U_z \end{bmatrix}, \quad (6)$$

and its orbital velocity  $V'_{\text{frg}}$ :

$$V'_{\text{frg}} = \sqrt{(\dot{x}'_{\text{frg}})^2 + (\dot{y}'_{\text{frg}})^2 + (\dot{z}'_{\text{frg}})^2} \quad (7)$$

The companion's orbital eccentricity,  $e'$ , is equal to

$$e' = \sqrt{1 + p' \left[ \left( \frac{V'_{\text{frg}}}{k} \right)^2 - \frac{2}{r_{\text{frg}}} \right]}, \quad (8)$$

where  $k$  is the Gaussian gravitational constant,

$$p' = \frac{\Re_{xy}^2 + \Re_{yz}^2 + \Re_{zx}^2}{k^2}, \quad (9)$$

and

$$\Re_{xy} = \begin{vmatrix} x_{\text{frg}} & y_{\text{frg}} \\ \dot{x}_{\text{frg}} & \dot{y}_{\text{frg}} \end{vmatrix}, \quad \Re_{yz} = \begin{vmatrix} y_{\text{frg}} & z_{\text{frg}} \\ \dot{y}_{\text{frg}} & \dot{z}_{\text{frg}} \end{vmatrix}, \quad \Re_{zx} = \begin{vmatrix} z_{\text{frg}} & x_{\text{frg}} \\ \dot{z}_{\text{frg}} & \dot{x}_{\text{frg}} \end{vmatrix}. \quad (10)$$

The companion's perihelion distance is  $q' = p'/(1+e')$  and its true anomaly  $v'_{\text{frg}}$  at  $t_{\text{frg}}$  is computed from

$$\sin v'_{\text{frg}} = \frac{\sqrt{p'}}{k r_{\text{frg}} e'} (x_{\text{frg}} \dot{x}'_{\text{frg}} + y_{\text{frg}} \dot{y}'_{\text{frg}} + z_{\text{frg}} \dot{z}'_{\text{frg}}),$$

$$\cos v'_{\text{frg}} = \frac{1}{e'} \left( \frac{p'}{r_{\text{frg}}} - 1 \right). \quad (11)$$

The eccentric anomaly at  $t_{\text{frg}}$ ,  $E'_{\text{frg}}$ , then comes out to be

$$E'_{\text{frg}} = 2 \arctan \left( \sqrt{\frac{1-e'}{1+e'}} \tan \frac{1}{2} v'_{\text{frg}} \right) \quad (12)$$

and the companion's perihelion time is finally derived from the relation

$$t'_\pi = t_{\text{frg}} - \frac{E'_{\text{frg}} - e' \sin E'_{\text{frg}}}{k} \left( \frac{q'}{1-e'} \right)^{\frac{3}{2}}. \quad (13)$$

Inserting from Equation (13) into (4), we obtain the temporal separation between the companion and the primary at perihelion,  $\Delta t_\pi$ , the quantity we were looking for.

<sup>4</sup> This is a right-handed orthogonal coordinate system, whose origin is the primary fragment, with the **R** axis pointing radially away from the Sun, the transverse **T** axis located in the primary's orbital plane, and the **N** axis normal to this plane.

Table 2

SEPARATION VELOCITY NEEDED FOR 76.274 DAYS GAP BETWEEN PERIHELION ARRIVAL TIMES OF C/1988 F1 AND C/1988 J1 (HELIOCENTRIC OSCULATING APPROXIMATION).

Parent comet's fragmentation			Separation velocity	
distance $r_{\text{frag}}$ (AU)	time before perihelion (yr)	(units of $P$ )	radial, $U_R$ (m s $^{-1}$ )	transverse, $U_T$ (m s $^{-1}$ )
200	-227.5	-0.0183	+4.30	+14.95
300	-431.2	-0.0346	+1.80	+7.76
400	-688.3	-0.0552	+0.94	+4.80
500	-1002.5	-0.0805	+0.56	+3.25
600	-1381.6	-0.1109	+0.35	+2.33
700	-1840.0	-0.1477	+0.23	+1.72
800	-2404.1	-0.1929	+0.16	+1.30
900	-3129.0	-0.2511	+0.11	+0.97
1000	-4176.3	-0.3352	+0.067	+0.70
1073.74 <sup>a</sup>	-6230.0	-0.5000	+0.034	+0.43
1000	-8283.7	-0.6648	+0.019	+0.28
900	-9331.0	-0.7489	+0.014	+0.22
800	-10055.9	-0.8071	+0.011	+0.18
700	-10620.0	-0.8523	+0.0094	+0.15
600	-11078.4	-0.8891	+0.0079	+0.13
500	-11457.5	-0.9195	+0.0066	+0.10
400	-11771.7	-0.9448	+0.0055	+0.082
300	-12028.8	-0.9654	+0.0045	+0.061
200	-12232.5	-0.9817	+0.0034	+0.040
100	-12381.4	-0.9937	+0.0023	+0.020

<sup>a</sup> Aphelion.

We note that, as byproducts of the computations, we also determined the effects of the companion's separation velocity on the perihelion distance,  $\Delta q = q' - q$ , and eccentricity  $\Delta e = e' - e$ . It is similarly possible to derive the separation velocity's effects on the remaining elements — the inclination, the longitude of the ascending node, and the argument of perihelion.

For C/1988 F1 and C/1988 J1 these computations show in Table 2 that a gap of 76 days in the time of arrival at perihelion is achievable with a separation velocity of  $\sim 1$  m s $^{-1}$  in the direction away from the Sun at a heliocentric distance of barely 400 AU, less than 700 yr before the 1987 perihelion. The same effect is also achievable with a separation velocity of 1 m s $^{-1}$  in the direction perpendicular to the radial direction in the orbital plane at a heliocentric distance of less than 900 AU some 3000 yr before the 1987 perihelion. Only the component of the separation velocity that is normal to the orbital plane cannot bring about a gap of this magnitude.

It is noted in Table 2 that, based on the radial component of the separation velocity, the fragmentation of the parent of C/1988 F1 and C/1988 J1 most probably occurred between 300 AU and 900 AU and not longer than 0.25 the orbital period before the 1987 perihelion, possibly as recently as the 14th century AD. If it had occurred much earlier, the radial component would have to have been more than an order of magnitude smaller than the transverse component, an unlikely scenario.

Since we neglected the planetary perturbations, we checked the accuracy of our results against a rigorous orbit-determination run. We integrated the set of elements of C/1988 F1 from Table 1 back in time to 1300 January 1, when the comet was 402.8 AU from the Sun, added exactly 1 m s $^{-1}$  to the radial component of its

Table 3

COMPARISON OF EFFECTS OF SEPARATION VELOCITY AT 402.8 AU FROM THE SUN AND PLANETARY PERTURBATIONS ON ORBITAL ELEMENTS OF COMET C/1988 F1 (EPOCH 1987 NOVEMBER 21.0 TT).

Increment in orbital element	Rigorous integration of orbit <sup>a</sup>	Separation velocity (m s $^{-1}$ ) alone		
		$U_R = +1.00$	$U_T = +5.02$	$U_N = +0.10$
$\Delta t_\pi$ (day)	+80.3136	+82.1665	+82.1665	+0.0007
$\Delta \omega$	+0°.0004	+0°.0029	-0°.2186	-0°.0125
$\Delta \Omega$	-0°.0038	0°.0000	0°.0000	+0°.0272
$\Delta i$	-0°.0025	0°.0000	0°.0000	-0°.0444
$\Delta q$ (AU)	+0.000073	+0.000003	+0.106562	+0.000001
$\Delta e$	+0.0000142	-0.0000044	-0.0001966	0.0000000

<sup>a</sup> Increments are the sums of an effect due to a separation velocity of  $U_R = +1.00$  m s $^{-1}$  and the planetary perturbations.

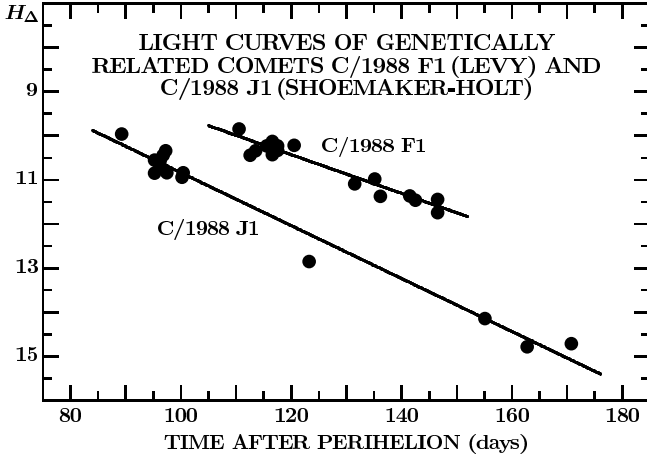
orbital velocity, and integrated forward in time to the epoch of 1987 November 21. The effects on the elements are shown in column 2 of Table 3. The perihelion passage came out to take place on 1988 February 18.2664 TT; the computations based on Equations (4)–(13) show a perihelion time of 1988 February 20.1137 TT, differing by  $\sim 2\%$  of the 80.3 day gap and indicating that the effect of planetary perturbations is almost two orders of magnitude *smaller* than the separation-velocity effect.

Table 3 summarizes the test results for the effects of separation-velocity not only on the perihelion time, but on the other elements as well. We note from columns 2 and 3 that the perihelion time  $t_\pi$  is the only orbital element for which the effect of the radial component of the separation velocity dominates the planetary perturbations. The tabulated values of  $\Delta \Omega$  and  $\Delta i$  are the net effects due to the perturbations, because no change can be triggered by a radial separation velocity in a direction off the orbital plane that these elements reflect.

In the penultimate column of Table 3 we list the orbital increments that would be triggered by a transverse component of the separation velocity generating the same effect in the perihelion time as a 1 m s $^{-1}$  radial component. Striking are the large changes in  $\omega$  and  $q$ , which are not observed. The last column shows the increments due to a small normal component of the separation velocity. The exceptional similarity of the angular elements and perihelion distance of the orbits of C/1988 F1 and C/1988 J1 (Table 1) suggests that the transverse and normal components of the separation velocity must have been very small compared to the radial component  $U_R$  and almost certainly much smaller than 0.1 m s $^{-1}$ .

### 2.1.2. Effects of a nongravitational acceleration

Our reconstruction of the light curves of C/1988 F1 and C/1988 J1 is shown in Figure 1, its caption describing the sources of brightness data used. The plot leaves no doubt that C/1988 F1, the leading fragment, was intrinsically brighter than C/1988 J1 by more than 1 mag; the latter object was also fading at a steeper rate, making the brightness ratio of C/1988 F1 to C/1988 J1 to increase with time. Thus, the relative positions of the two fragments in the orbit are consistent with the fainter (and presumably less massive) one having been decelerated relative to the brighter (and presumably more massive) one. Accordingly, the temporal separation of



**Figure 1.** The light curves of the genetically related comets C/1988 F1 and C/1988 J1. Plotted against time (reckoned from perihelion of each comet) are the total magnitudes  $H_{\Delta}$ , normalized to a unit geocentric distance, approximately corrected for personal and instrumental effects, and reduced to the photometric system of D. Levy, who made most of the plotted observations. The data were collected from a number of issues of the *International Comet Quarterly* and from the *Minor Planet Circulars* [only a selection of magnitudes referred to as total (T)].

76 days between both comets could have been affected to a degree by a differential nongravitational acceleration. We now investigate this problem in more detail.

We begin by assuming that the common parent of comets C/1988 F1 and C/1988 J1 had moved in a gravitational orbit identical with that of C/1988 F1, fragmenting on its way from aphelion to perihelion at a time  $t_{\text{frg}}$ , when it was at a heliocentric distance  $r_{\text{frg}}$  (Section 2.1.1). If the primary fragment, C/1988 F1, arrived at perihelion at time  $t_{\pi}$ , the length of the orbital arc that it traveled between  $t_{\text{frg}}$  and  $t_{\pi}$  is equal to

$$\ell = \int_{t_{\text{frg}}}^{t_{\pi}} V_{\text{gr}}(t) dt, \quad (14)$$

where  $V_{\text{gr}}(t)$  is the orbital velocity of the primary's gravitational motion at time  $t$ . We now assume that the orbital motion of the companion C/1988 J1 has been — ever since the secession from the parent (at a zero separation velocity) — affected by a nongravitational acceleration pointing in a general direction against the orbital motion. During the period of time from  $t_{\text{frg}}$  to  $t_{\pi}$  the orbital arc traveled by the companion is accordingly shorter by  $\Delta\ell$ ,

$$\ell - \Delta\ell = \int_{t_{\text{frg}}}^{t_{\pi}} V_{\text{ng}}(t) dt, \quad (15)$$

where  $V_{\text{ng}}(t)$  is the orbital velocity of the companion's nongravitational motion. Let  $V_{\text{ng}}(t)$  be a sum of a gravitational part, which is equal to  $V_{\text{gr}}(t)$ , the orbital velocity of the primary fragment from Equation (14), plus the nongravitational part,  $\Delta V_{\text{ng}}(t) = V_{\text{ng}}(t) - V_{\text{gr}}(t) < 0$ , which is equal to an effect of a nongravitational acceleration in the direction of the orbital-velocity vector integrated from  $t_{\text{frg}}$  to  $t$ . Subtracting Equation (15) from Equation (14), introducing a ratio  $z = q/r$  (where  $r$  is the heliocentric distance at time  $t$ ) as a new integration variable, and approximating the gravitational orbit by a

parabola, we readily find

$$\Delta\ell = -\frac{q^{\frac{3}{2}}}{k\sqrt{2}} \int_{z_{\text{frg}}}^{z_{\star}} \Delta V_{\text{ng}}(z) z^{-\frac{5}{2}} (1-z)^{-\frac{1}{2}} dz, \quad (16)$$

where  $k = 0.0172021 \text{ AU}^{\frac{3}{2}} \text{ day}^{-1}$  is the Gaussian gravitational constant,  $q$  is again the perihelion distance of the primary's orbit,  $z_{\text{frg}} = q/r_{\text{frg}}$ , and  $z_{\star} = q/r_{\star}$ , where  $r_{\star}$  is the heliocentric distance of the companion at the time the primary is at perihelion. The companion still has  $\Delta t_{\pi} = 76.274$  days to go to its perihelion and the condition that  $z_{\star}$  should satisfy is

$$\Delta t_{\pi} = \frac{q^{\frac{3}{2}} \sqrt{2}}{3k} z_{\star}^{-\frac{3}{2}} (1+2z_{\star}) \sqrt{1-z_{\star}}. \quad (17)$$

Next we consider separately the radial,  $j_{\text{R}}$ , and transverse,  $j_{\text{T}}$ , components of the nongravitational acceleration in the orbital plane of the **RTN** right-handed coordinate system (Section 2.1.1). In the Marsden et al.'s (1973) Style II formalism, they are written in the following form:

$$\begin{bmatrix} j_{\text{R}}(r) \\ j_{\text{T}}(r) \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} g(r), \quad (18)$$

where  $g(r)$  is the standard empirical nongravitational law that the formalism employs,

$$g(r) = 0.1113 (r/r_0)^{-2.15} [1 + (r/r_0)^{5.093}]^{-4.6142}, \quad (19)$$

with a scaling distance  $r_0 = 2.808 \text{ AU}$  appropriate for an acceleration driven by the sublimation of water ice. The law is normalized to  $g(1 \text{ AU}) = 1$ , so that the parameters  $A_1$  and  $A_2$  are the radial and transverse accelerations at a unit heliocentric distance.

The integrated contribution from the nongravitational acceleration to the magnitude of the orbital-velocity vector,  $\Delta V_{\text{ng}}$  (now reckoned positive in the direction opposite the orbital motion), is expressed, in a parabolic approximation, for any time  $t$  ( $t_{\text{frg}} < t < t_{\pi}$ ) along a preperihelion orbital arc by

$$\Delta V_{\text{ng}}(t) = A_1 \int_{t_{\text{frg}}}^t g[r(t)] \sqrt{1-z} dt \quad (20)$$

from the radial component and

$$\Delta V_{\text{ng}}(t) = -A_2 \int_{t_{\text{frg}}}^t g[r(t)] \sqrt{z} dt \quad (21)$$

from the transverse component. Inserting Equations (20) and (21) into Equation (16) and again replacing time with the ratio  $z$  as an integration variable, we find in the case that the effect is due entirely to the radial component of the nongravitational acceleration:

$$\Delta\ell = \frac{A_1 q^3}{2k^2} \int_{z_{\text{frg}}}^{z_{\star}} z^{-\frac{5}{2}} (1-z)^{-\frac{1}{2}} dz \int_{z_{\text{frg}}}^z g\left(\frac{q}{\zeta}\right) \zeta^{-\frac{5}{2}} d\zeta \quad (22)$$

and similarly in the case that it is due entirely to the transverse component:

$$\Delta\ell = -\frac{A_2 q^3}{2k^2} \int_{z_{\text{frg}}}^{z_{\star}} z^{-\frac{5}{2}} (1-z)^{-\frac{1}{2}} dz \int_{z_{\text{frg}}}^z g\left(\frac{q}{\zeta}\right) \zeta^{-2} (1-\zeta)^{-\frac{1}{2}} d\zeta. \quad (23)$$

Table 4

ORBITAL ELEMENTS OF COMETS C/1988 A1 (LILLER), C/1996 Q1 (TABUR), AND C/2015 F3 (SWAN) (EQUINOX J2000.0).

Quantity/Orbital element	Comet C/1988 A1 <sup>a</sup>	Comet C/1996 Q1	Comet C/2015 F3
Osculation epoch (TT)	1988 Mar. 20.0	1996 Nov. 13.0 <sup>b</sup>	2015 Feb. 27.0
Time of perihelion passage $t_\pi$ (TT)	1988 Mar. 31.11442	1996 Nov. 3.53123 $\pm$ 0.00274	2015 Mar. 9.35613 $\pm$ 0.00077
Argument of perihelion $\omega$	57° 38762	57° 41221 $\pm$ 0° 00314	57° 56668 $\pm$ 0° 00121
Longitude of ascending node $\Omega$	31° 51540	31° 40011 $\pm$ 0° 00102	31° 63895 $\pm$ 0° 00042
Orbit inclination $i$	73° 32239	73° 35626 $\pm$ 0° 00119	73° 38679 $\pm$ 0° 00023
Perihelion distance $q$	0.8413332	0.8398052 $\pm$ 0.0000139	0.8344515 $\pm$ 0.0000121
Orbital eccentricity $e$	0.9965647	0.9986821 $\pm$ 0.0001399	0.9964470 $\pm$ 0.0000348
Orbital period (yr) $\left\{ \begin{array}{l} \text{osculation} \\ \text{original}^c \end{array} \right.$	$3832.7 \pm 9.4$ 2933.0	16 100 $\pm$ 2600 10 500	3599 $\pm$ 53 3300
Orbital arc covered by observations	1988 Jan. 12–1988 July 14	1996 Aug. 21–1996 Oct. 18	2015 Mar. 24–2015 May 28
Number of observations employed	100	199	283
Root-mean-square residual	$\pm 0''.8$	$\pm 1''.0$	$\pm 0''.8$
Orbit-quality code <sup>d</sup>	1B	2B	2A
Reference	Green (1988)	JPL database (Orbit 15) <sup>e</sup>	Williams (2015)

<sup>a</sup> The errors of the elements other than the orbital period are unavailable.<sup>b</sup> After integration of the elements from an initial nonstandard epoch of 1996 Sept. 18.0 TT.<sup>c</sup> With respect to the barycenter of the Solar System; rigorous orbit integration gives for  $t_\pi$  at the previous return the date of –944 June 20.<sup>d</sup> Following the classification system introduced by Marsden et al. (1978).<sup>e</sup> See <http://ssd.jpl.nasa.gov/sbdb.cgi>.

These expressions determine the parameters  $A_1$  and  $A_2$ , since the traveled orbital-arc length  $\Delta\ell$  is, in an adequate parabolic approximation, a function of only  $q$  and  $z_*$ :

$$\Delta\ell = k \int_{t_\pi}^{t'_\pi} \sqrt{\frac{2}{r}} dt = q \int_{z_*}^1 \frac{dz}{z^2 \sqrt{1-z}} = 2q \int_0^{\sqrt{z_*^{-1}-1}} \sqrt{1+x^2} dx$$

$$= q \left( \frac{\sqrt{1-z_*}}{z_*} + \log_e \frac{1+\sqrt{1-z_*}}{\sqrt{z_*}} \right). \quad (24)$$

With  $\Delta t_\pi = 76.274$  days and  $q = 1.17418$  AU, we find  $z_* = 0.70860$  (or  $r_* = 1.65705$  AU) from Equation (17) and  $\Delta\ell = 1.60358$  AU from Equation (24).

The nongravitational parameters  $A_1$  and  $A_2$  are now functions of only the heliocentric distance at fragmentation,  $r_{\text{frag}}$ . There is no measurable contribution from the heliocentric distances much greater than the scaling distance  $r_0$  [Equation (19)], as the nongravitational law  $g(r)$  falls off precipitously at those distances. This expectation is fully corroborated by numerical integration of Equations (22) and (23), resulting in the nongravitational parameters  $A_1 = +0.00642$  AU day<sup>−2</sup> and  $A_2 = -0.00575$  AU day<sup>−2</sup> for any  $r_{\text{frag}} \gtrsim 3.6$  AU. For smaller  $r_{\text{frag}}$  the magnitudes of  $A_1$  and  $A_2$  are greater still. The inferred minimum values  $|A_1|$  and  $|A_2|$  are found to exceed the gravitational acceleration of the Sun at 1 AU by a factor of  $\sim 20$  or more and are meaningless. We conclude that the nongravitational effects in the motions of C/1988 F1 and C/1988 J1 cannot account for the gap of  $\sim 76$  days between their perihelion times.

These results, too, were confronted with rigorous orbital computations, which showed in the first place that a positive radial nongravitational acceleration ( $A_1 > 0$ ) would in fact make the companion C/1988 J1 pass through perihelion *before* the primary C/1988 F1. This is so because by reducing the orbital velocity, this acceleration increases the perihelion distance and significantly shortens the orbital period.

With reference to a negative radial nongravitational acceleration, the rigorous orbital computations suggested

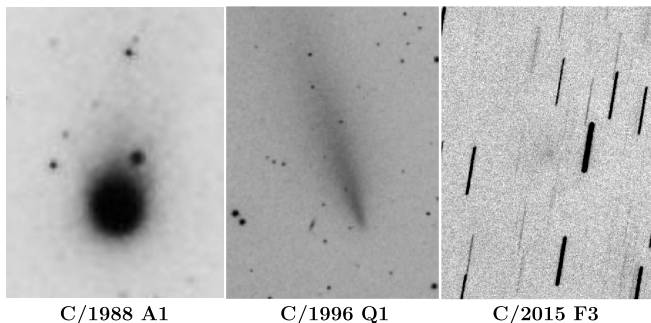
that if a fragmentation event occurred, for example, at 402.8 AU before perihelion, an acceleration described by  $A_1 = -10^{-6}$  AU day<sup>−2</sup>, about 0.3% of the Sun's gravitational acceleration that would delay the perihelion time by merely 0.044 day, should already be judged as unacceptably high in magnitude, because it would — contrary to the trivial differences between the orbital elements of C/1988 F1 and C/1988 J1 — increase the argument of perihelion by 0° 137 and decrease the perihelion distance by 0.0014 AU. If nearly comparable with the Sun's gravitational acceleration, these nongravitational effects would make the companion's orbit strongly hyperbolic.

Similarly, the transverse nongravitational acceleration described by  $A_2 = -10^{-6}$  AU day<sup>−2</sup>, which would delay the passage through perihelion by 0.24 day, is already unacceptably high in magnitude, because it would increase the argument of perihelion by 0° 227 and reduce the perihelion distance by 0.0018 AU. If nearly comparable with the Sun's gravitational acceleration, these nongravitational effects would transform the companion into a short-period comet of a small perihelion distance.

## 2.2. A Trio of Genetically Related Comets

Another comet that was discovered in early 1988 was C/1988 A1 (Liller), a fairly, but not extraordinarily, active one. We had to wait for more than 8 years to appreciate its special status. In August of 1996, a newly discovered comet C/1996 Q1 (Tabur) turned out to have an orbit nearly as similar to that of C/1988 A1 as was the orbit of C/1988 J1 to that of C/1988 F1. And just months before this writing, yet another object was discovered, C/2015 F3 (SWAN), also with an orbit nearly identical to those of C/1988 A1 and C/1996 Q1. The sets of orbital elements for the three comets are compared in Table 4. It is unfortunate that their quality is very uneven. The best determined orbit is that of C/1988 A1, which was astrometrically measured both before and after perihelion and the observed orbital arc was just about six months. Although the formal errors of the elements were not — except for the orbital period (or the semi-major axis) — published, their uncertainty is not greater

## TRIO OF GENETICALLY RELATED COMETS



**Figure 2.** Dramatically different appearance of three genetically related comets. All three images are 14' along a diagonal. The image of comet C/1988 A1 is a 12-minute exposure obtained by M. Oates from the suburbs of Manchester, England, with a 6.7-cm f/4.5 camera on 1988 April 23.974 UT (see <http://www.manastro.co.uk/members/contrib/maotes/metcomet/liller88.htm>), about 24 days after perihelion. The comet was 1.31 AU from the Earth and 0.95 AU from the Sun. — The image of comet C/1996 Q1 is a 5-minute exposure by H. Mikuz with a 36-cm f/6.8 Schmidt-Cassegrain telescope of the Črni Vrh Observatory near Idrija, Slovenia. At the time of observation, 1996 November 9.709 UT (see <http://www.observatorij.org/>), the comet was 6 days past perihelion, 0.99 AU from the Earth and 0.85 AU from the Sun. — The image of comet C/2015 F3 is a sum of ten 3-minute exposures obtained by G. Masi remotely with a 43-cm f/6.8 robotic unit of the Virtual Telescope Project, Ceccano, Italy, on 2015 May 18.944 UT (see <http://www.virtualtelescope.eu/2015/05/19/comet-c2015-f3-swan-image-18-2015>), some 70 days after perihelion. At this time the comet was 1.16 AU from the Earth and 1.48 AU from the Sun. (Credits: M. Oates; Črni Vrh Observatory; Virtual Telescope Project, Bellatrix Astronomical Observatory.)

than a few units of the last or penultimate decimal place. The second best determined orbit is that of C/2015 F3, whose story of discovery and follow-up observation is peculiar (Green 2015). Although its SWAN Lyman-alpha images were detected as early as 2015 March 3, 6 days before perihelion, the astrometric and brightness observations did not commence until March 24, more than two weeks after perihelion. The delay notwithstanding, it was astrometrically measured over a period of slightly more than two months. By contrast, C/1996 Q1, whose orbit is least well determined, was discovered more than 10 weeks before perihelion, but the astrometric observations had to be terminated some eight weeks later, still before perihelion, because of the loss of the nuclear condensation. The orbital arc's length covered by the astrometry was a few days short of two months.

In spite of being fragments of a common parent, the three comets had a very different appearance, morphology, and behavior. This is illustrated by both the sample images in Figure 2 and by the light curves in Figure 3.

Comet C/1988 A1, presented in Figure 2 as it looked near its peak intrinsic brightness, displays a coma of at least 3', equivalent to 170 000 km, in diameter and appears to be healthy and active. The light curve in Figure 3 shows that its post-perihelion fading was, at least at heliocentric distances smaller than 2 AU, substantially less steep than its preperihelion brightening and that the intrinsic brightness peaked some three weeks after perihelion. Overall, the light curve was fairly smooth, again especially within 2 AU of the Sun.

The behavior of C/1996 Q1 was, in several respects, contrary to that of C/1988 A1. The image of C/1996 Q1 in Figure 2, from November 9, six days after perihelion,

consists of a headless long narrow tail, without any nuclear condensation, which explains the termination of astrometric observations long before this image was taken. The light curve in Figure 3 is quite erratic, displaying at least three outbursts between 40 and 20 days before perihelion. The first of them is seen to have begun more than 50 days before perihelion and for more than 10 days in this span of time C/1996 Q1 was intrinsically *brighter* than C/1988 A1 at the same heliocentric distance (note the shift in the brightness scale in Figure 3). The last of the three outbursts was accompanied by major morphological changes in the comet's head, with the condensation disappearing within a few days following the event's peak. It was at this time, about 16 days before perihelion, that all astrometric observations terminated. The light curve shows a continuous steep decline of brightness ever since the peak of the third outburst. The last confirmed magnitude estimates were obtained about two weeks after perihelion, but extremely faint “ghost” images of the comet's residual tail were marginally detected<sup>5</sup> as late as mid-January of 1997.

Comparison of C/2015 F3 with C/1996 Q1 suggests only one common trait: both comets were fading much more rapidly than C/1988 A1, obviously the primary, most massive fragment of their common parent. Otherwise C/2015 F3 differed from C/1996 Q1 significantly. First of all, C/2015 F3 survived perihelion seemingly intact. In fact, relative to perihelion, this comet was not even discovered in the SWAN images (Green 2015) by the time C/1996 Q1 had already disintegrated. Second, the light curve of C/2015 F3 is in Figure 3 located above the light curve of C/1996 Q1. Third, no prominent tail was observed to have survived the head of C/2015 F3, as illustrated in Figure 2 by one of the images in its advanced phase of evolution. It was not until some 50 or so days after perihelion that this comet began to lose its nuclear condensation, which resulted in an accelerated fading (Figure 3). And, fourth, there is no evidence that this rapid fading was triggered by an outburst, as it was in the case of C/1996 Q1.

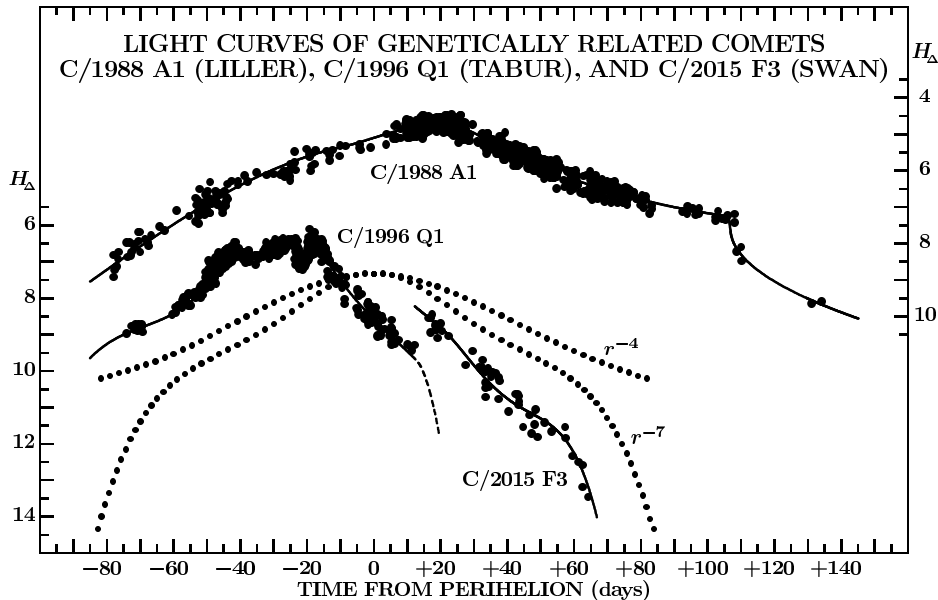
### 2.2.1. Fragmentation of the parent

With the previous return to perihelion computed to have occurred in 945 BCE (note c in Table 4), we find that both C/1996 Q1 and C/2015 F3 are likely to have separated from C/1988 A1 in their common parent's fragmentation event or events in a general proximity of that perihelion. Equation (2) implies that the gaps of 3139.42 days between the perihelion times of C/1988 A1 and C/1996 Q1 and 9839.24 days between C/1988 A1 and C/2015 F3 require separation velocities (in m s<sup>-1</sup>) along the orbital-velocity vector of, respectively,

$$\begin{aligned}\Delta V &= 0.092\sqrt{r_{\text{frg}}/q} \quad \text{for C/1996 Q1 vs C/1988 A1,} \\ \Delta V &= 0.289\sqrt{r_{\text{frg}}/q} \quad \text{for C/2015 F3 vs C/1988 A1.}\end{aligned}\quad (25)$$

Since the direction of the separation velocity vector  $\mathbf{V}_{\text{sep}}$  generally differs from the direction of the orbital-velocity vector,  $\mathbf{V}_{\text{frg}}$ , at the time of fragmentation, it is always  $V_{\text{sep}} = |\mathbf{V}_{\text{sep}}| \geq \Delta V$ . If  $\theta$  is an angle between the two vectors, then  $\cos\theta = \Delta V/V_{\text{sep}}$  and a statistically aver-

<sup>5</sup> See, e.g., <http://www.observatorij.org>.



**Figure 3.** Light curves of three genetically related comets C/1988 A1, C/1996 Q1, and C/2015 F3. Plotted against time (reckoned from perihelion of each comet) are the total magnitudes  $H_{\Delta}$ , normalized to a unit geocentric distance, approximately corrected for personal and instrumental effects, and reduced to the photometric system of experienced visual observers. The magnitude scale on the right applies to C/1988 A1, the scale on the left, shifted by 2 magnitudes upward, to C/1996 Q1 and C/2015 F3. The data were collected from issues of the *International Comet Quarterly*; for C/2015 F3, from the ICQ website and the *Minor Planet Circulars* [only selected magnitudes referred to as total (T)]. The light curve of C/1996 Q1 does not show an unconfirmed observation by a single observer, made 20 days after perihelion (Marsden 1996), which was nominally inconsistent with a nearly simultaneous nondetection by another observer and with the comet’s systematic fading trend. The dotted curves show the variations following the laws  $r^{-4}$  and  $r^{-7}$ , respectively.

aged angle  $\langle\theta\rangle$  is given by integrating over a hemisphere centered on the direction of the orbital-velocity vector,

$$\frac{\pi}{2} \langle\cos\theta\rangle = \int_0^{\frac{\pi}{2}} \sin\theta \cos\theta d\theta = \frac{1}{2}, \quad (26)$$

so that  $\langle\cos\theta\rangle = 1/\pi$  and a statistically averaged separation velocity  $\langle V_{\text{sep}}\rangle = \pi\Delta V$ .

Fragmentation events are most likely to occur at, but are not limited to, times not too distant from the perihelion time. Considering, for example, that the parent comet of C/1988 A1, C/1996 Q1, and C/2015 F3 split at a time of up to a year around perihelion, equivalent to heliocentric distances of less than  $\sim 5$  AU, an overall range of implied velocities  $\Delta V$  for the two companions is from  $\sim 0.1$  to  $\sim 0.7$  m s $^{-1}$  and a range for the separation velocities  $V_{\text{sep}}$  is from  $\sim 0.3$  to  $\sim 2$  m s $^{-1}$ , in perfect harmony with expectation (Section 2.1.1).

#### 2.2.2. Comparison of C/1988 A1 and C/1996 Q1 in terms of their physical properties

The manifest sudden termination of activity of comet C/1996 Q1 about 16 days before perihelion is independently documented by the water production data listed in Table 5. The numbers also show that the production of water from C/1996 Q1 was apparently stalling for at least 10 days before its termination, that 10 days before perihelion the water production of C/1988 A1 was more than 10 times as high as that of C/1996 Q1, and that 43 days after perihelion C/1988 A1 produced about as much water as C/1996 Q1 19 days before perihelion.

The image of C/1996 Q1 in Figure 2 is one of three that Fulle et al. (1998) used to study the comet’s dust tail. The employed method (Fulle 1989) allowed them to determine, among others, the production rate of dust as

a function of time. For a bulk density of dust particles of 1 g cm $^{-3}$  and a product of their geometric albedo and phase function (with the phase angles of  $65^{\circ}$  to  $75^{\circ}$  at the imaging times) of 0.02, Fulle et al.’s effort resulted in four similar dust production curves, one of which is plotted in Figure 4. Because the employed method has a tendency to smooth short-term variations, minor outbursts are suppressed. This is the reason why the curve in the figure does not show the steep increase in the amount of dust in the comet’s atmosphere around September 14, 50 days before perihelion (an event that was called attention to by Lara et al. 2001), and the terminal outburst one month later shows up as a relatively small bulge.

Comparison with the water production data in Figure 4 suggests a water-to-dust mass production rate ratio of  $\sim 15$ , so that C/1996 Q1 was an exceptionally dust-poor comet. Integrating the dust production curve from 100 days before perihelion to 25 days after perihelion, Fulle et al. (1998) determined that over this span of time the comet lost  $1.2 \times 10^{12}$  g of dust. An exponential extrapolation over the rest of the orbit (outlined by the dashed curve in Figure 4) does not increase the mass to more than  $1.4 \times 10^{12}$  g. Assumptions of an overall constant water-to-dust mass production rate ratio (15:1) and of water dominance among volatile species [cf. Womack & Suswal’s (1996) unsuccessful search for carbon monoxide] led to a conservative lower limit of  $2 \times 10^{13}$  g for this comet’s total mass in the case of complete disintegration. Possible survival of a limited number of boulder-sized fragments cannot significantly affect this estimate.

Fulle et al. (1998) alleged that the nucleus of comet C/1996 Q1 did not fragment and that its activity ceased due to mantle formation, seasonal variations, or depletion of volatile species, including water. These authors



**Table 5**  
WATER PRODUCTION RATES FROM COMETS C/1988 A1 (LILLER) AND C/1996 Q1 (TABUR).

Comet	Time from perihelion (days)	Distance from Sun (AU)	Production rate of H <sub>2</sub> O (10 <sup>5</sup> g/s)	Method used <sup>a</sup>	Active area <sup>b</sup> (km <sup>2</sup> )	Reference
C/1988 A1	-10 <sup>c</sup>	0.86	35	OH (radio)	5.5	Crovisier et al. (2002)
	+40	1.12	13.7	OH (UVB)	3.8	A'Hearn et al. (1995)
C/1996 Q1	-30 <sup>d</sup>	1.01	16	OH (radio)	3.5	Crovisier et al. (2002)
	-28	0.99	10	H <sub>2</sub> O <sup>+</sup> (red)	2.1	Jockers et al. (1999)
	-26 <sup>e</sup>	0.97	14	OH (radio)	2.8	Crovisier et al. (2002)
	-23 <sup>f</sup>	0.94	11	OH (radio)	2.1	Crovisier et al. (2002)
	-19	0.92	12.6	H (EUV)	2.3	Mäkinen et al. (2001)
	-10 <sup>g</sup>	0.86	<3	OH (radio)	<0.5	Crovisier et al. (2002)

<sup>a</sup> Water production rate assumed 1.1 times hydroxyl production rate; OH (radio) = from observation of hydroxyl emission at 18 cm with a radiotelescope at Nançay; OH (UVB) = from narrow-band photometry of hydroxyl emission at 3085 Å; H<sub>2</sub>O<sup>+</sup> (red) = from column-density maps of ionized water emission at 6153 Å; H (EUV) = from maps of Lyman-alpha emission of atomic hydrogen at 1216 Å taken with the SWAN imager on board SOHO.

<sup>b</sup> Effective sublimation area, when the Sun is at zenith.

<sup>c</sup> Average from data taken between 1988 March 17 and 24.

<sup>d</sup> Average from data taken between 1996 October 2 and 5.

<sup>e</sup> Average from data taken between 1996 October 7 and 9.

<sup>f</sup> Average from data taken between 1996 October 10 and 12.

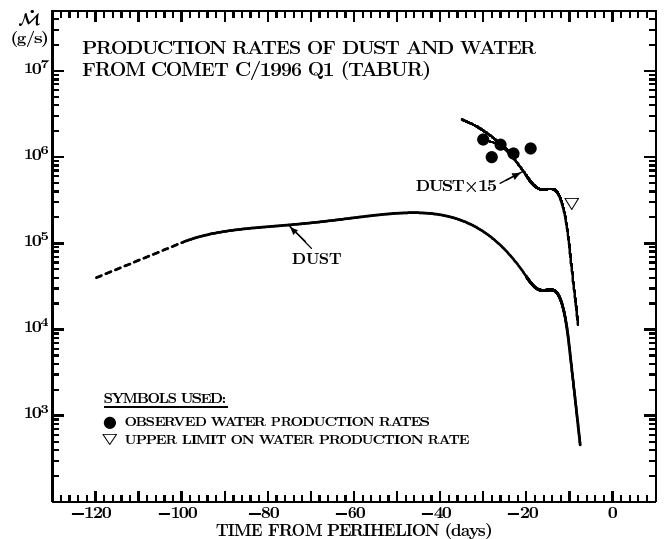
<sup>g</sup> Average from data taken between 1996 October 20 and 27.

argued that the fairly high water production rate in the first half of October, a month before perihelion (cf. Table 5), suggested the nuclear diameter was greater than 0.7 km. We remark that the implied bulk density would then have been close to 0.1 g cm<sup>-3</sup>. Contrary to Fulle et al. (1998), Wyckoff et al. (2000) concluded from their observations with large telescopes that the nucleus probably began to fragment around October 7, some four weeks before perihelion, and that during the next several weeks it completely dissipated.

Because the same method of dust-tail analysis was also applied to C/1988 A1 (Fulle et al. 1992), it is possible to compare the two genetically related comets in terms of dust production. The images of C/1988 A1 selected for this study were taken with a 106-cm Cassegrain telescope at the Hoher List Observatory in mid-May 1988 (Rauer & Jockers 1990) and the adopted product of the geometric albedo and the phase function was 0.06, that is, three times higher than for C/1996 Q1, while the phase angle was 50°. Normalizing to the same geometric albedo, the results show for C/1988 A1 a dust-mass loss of  $1.4 \times 10^{14}$  g integrated between 200 days before perihelion and 40 days after perihelion; the total loss of dust over one revolution about the Sun can be estimated at more than  $2 \times 10^{14}$  g. Keeping in mind the factor of three, comparison of the dust production curve with the water production 10 days before perihelion (Table 5) suggests a water-to-dust mass production rate ratio of about 0.2. Similarly, at the time of the post-perihelion water production data point in the table, the ratio is about 0.3. Thus, with grains as dark as in C/1996 Q1, C/1988 A1 was a truly dust-rich comet, about 60 times dustier than C/1996 Q1, and its total loss of mass per revolution was near  $3 \times 10^{14}$  g, some 15 times higher than C/1996 Q1's.

It should be remarked that a strikingly different gas-to-dust production ratio for the two genetically related comets was noticed by Kawakita et al. (1997) after they compared the results of their spectroscopic observations of C/1996 Q1, made between 1996 September 14 and October 16, with the results for C/1988 A1 published

earlier by Baratta et al. (1989) and by A'Hearn et al. (1995). Because the data on the water and dust production of C/1996 Q1 (Crovisier et al. 2002; Mäkinen et al. 2001; Fulle et al. 1998) were not yet available, Kawakita et al. could determine this comet's gas-to-dust ratio only very approximately. From a continuum flux at 5200–5300 Å in their spectra they calculated the dust production's  $Af\rho$  proxy (A'Hearn et al. 1984) and from C<sub>2</sub>( $\Delta v = 0$ ) band the production rate  $Q(C_2)$ . They found



**Figure 4.** Dust and water production from C/1996 Q1 as a function of time. The thick curve is one of four similar models of dust emission, derived by Fulle et al. (1998) from their analysis of three CCD images taken by H. Mikuž on 1996 October 31 and on November 2 and 9 (see footnote earlier in Section 2.2). The employed method has a tendency to smooth short-term variations in the production rate of dust, so that minor outbursts are suppressed. The dashed part of the curve is an exponential extrapolation of dust production for a period of time ending 100 days before perihelion. The water production rates from Table 5 (solid circles; the three of them that are connected together come from the radio observations) and an upper limit on the production rate (an open triangle) are consistent with the thin curve, which represents a mass production rate 15 times higher than the dust rate.

that in the same range of heliocentric distance the ratio of  $Q(C_2)/Afp$  for C/1996 Q1 was four times greater than for C/1988 A1. Employing Fink & DiSanti's (1990) technique for deriving the water production rate from the intensity of the forbidden  $[OI]^1D$  line of atomic oxygen at 6300 Å, Kawakita et al. computed the ratio  $Q(H_2O)/Afp$  for C/1996 Q1 at the same heliocentric distance at which it was known for C/1988 A1 from A'Hearn et al. (1995), found that for C/1996 Q1 the ratio came out to be  $\sim 15$  times higher; they concluded that the nucleus of the parent comet must have been strongly inhomogeneous.<sup>6</sup>

The issue of whether the uneven dust content in the two comets correlates with the chemical composition of volatile species was addressed by Turner & Smith (1999). Comparing their own results for C/1996 Q1 from 1996 September 19–20 with those by A'Hearn et al. (1995) for C/1988 A1, they showed that the production ratios  $C_2/CN$ ,  $C_3/CN$ , and  $NH/CN$  were practically identical. The only exception was the abundance of OH relative to these molecules, which in C/1996 Q1 was an order of magnitude *lower*. Turner & Smith admitted that their OH results, derived from the observations of the (0-0) band at 3085 Å, may have been burdened by large errors.<sup>7</sup> Ignoring this discrepancy, the authors arrived at a conclusion that their results are consistent with a homogeneous composition for the ices in the nucleus of the parent comet.

Mineralogical comparison of the dust content in the comets C/1988 A1 and C/1996 Q1 is unfortunately not possible because the necessary data are not available for the first object. Examining the spectrophotometric data for C/1996 Q1 in the thermal infrared between 7.6  $\mu m$  and 13.2  $\mu m$ , Harker et al. (1999) found that a mineralogical model that best fits the spectra obtained on October 8–10, more than three weeks before perihelion, is a mix of grains of crystalline olivine 2  $\mu m$  in diameter and amorphous pyroxene 6  $\mu m$  in diameter. On the other hand, Turner & Smith (1999) allowed for the possible existence of gradually destroyed grains containing water ice in order to explain the blue color of the continuum in the optical spectrum at small distances from the nucleus on its sunward side.

It is too early to compare C/1988 A1 and C/1996 Q1 with C/2015 F3. However, the very facts that this new member of the group was first spotted due to its Lyman-alpha images of atomic hydrogen but appeared fairly faint and with no prominent dust tail to the ground observers suggests that, like C/1996 Q1, this comet was most probably water-rich and dust-poor.

<sup>6</sup> Kawakita et al. (1997) did not publish the value of the water production rate that they obtained at 1.15 AU before perihelion, which is the reason why we do not plot their result in Figure 4. From their ratio of the water production rate to  $Afp$  they inferred a gas-to-dust mass ratio of 8.5, consistent, within a factor of two, with the fit in Figure 4.

<sup>7</sup> Errors greatly exceeding one order of magnitude are certainly implied by comparison not only with the water production rates based on a variety of October observations, but also with the already discussed results by Kawakita et al. (1997), who began to make their spectroscopic observations five days before Turner & Smith's run. The latter's hydroxyl production rates, equivalent to water production rates of  $2.3\text{--}3.4 \times 10^4 \text{ g s}^{-1}$  on September 19–20 are about an order of magnitude lower than the dust production rate at the time, while the water production derived by Kawakita et al. was very much above the dust curve. For this reason, Turner & Smith's inferred water data points are not plotted in Figure 4.

### 3. THE LICK OBJECT OF 1921 AUGUST 7

Equipped with much information on the genetically related comet pairs and groups, we are now ready to attack the problem of the celebrated 1921 Lick object, whose detection caused much world-wide stir for a long time after the initial report was issued but whose identity remains a mystery to this day.

The story began with a party that gathered at the residence of W. W. Campbell, then Director of the Lick Observatory, on Mount Hamilton to watch the colorful setting Sun on 1921 August 7. As the Sun's disk was passing through the horizon haze, a member of the party, which also included H. N. Russell, asked a question on the identity of a bright star-like body 6 solar diameters (1 solar diameter at the time = 31'.55) to the east of the Sun. As subsequent scrutiny by Campbell and Russell eliminated the suspected bright planets, the two astronomers agreed that a short message should be dispatched at once to the Harvard College Observatory for immediate dissemination (Campbell 1921a, 1921b) and expanded reports of the phenomenon prepared later for scientific journals (Campbell 1921c, 1921d; Russell 1921).

Stellar even when briefly viewed through binoculars, brighter than Venus would have been if seen in the same position and circumstances, and sharing the Sun's motion in the sky, the object was located about 3° east and 1° south of the Sun (Campbell 1921c), at R.A. = 9<sup>h</sup> 22<sup>m</sup>, Dec. = +15°5 (Campbell 1921d). While no equinox was explicitly stated, the equinox in use at the time was generally 1900.0. The quoted coordinates put the Lick object ahead of the Sun by 3°.2 in right ascension and below the Sun by 0°.9 in declination, in slightly better agreement with the offset estimate in declination (Campbell 1921c) than would the same coordinates at the equinox of the date. As for the observation time, we find that at the top of Mount Hamilton the Sun set at 3:14 UT (August 8), not 2:50 UT as claimed by Pearce (1921). The time derived by us includes the effects of the observing site's elevation above sea level and atmospheric refraction and refers to the last contact with the true horizon.

The object was unsuccessfully searched for at Lick both at sunrise and sunset on August 8 and at sunrise on August 9 (Campbell 1921d). Because of its high galactic latitude ( $\sim 40^\circ$ ), the object was thought to be more probably the head of a comet than a nova.<sup>8</sup> However, we ruled out a member of the Kreutz sungrazer system, because at this time of the year it should approach the Sun from the southwest.<sup>9</sup> Subsequently we learned that a Kreutz sungrazer was eliminated as a potential candidate long ago (Ashbrook 1971; Baum 2007).

Although Campbell and his Lick party were not — as described below — the first to detect the object, they were the first to report it. This is why it is often referred to as the 1921 Lick object.

A number of reports appeared in worldwide response, but only a few of them included helpful temporal and positional information. We next list the more interesting observations in the order of decreasing relevance.

<sup>8</sup> An additional argument is that it is extremely unlikely for a nova to flare up at a position nearly coinciding with that of the Sun. On the other hand, the brightness of comets is known to increase dramatically near the Sun.

<sup>9</sup> See <http://www.rkracht.de/soho/c3kreutz.htm>.

An elaborate account of the circumstances of one such observation, originating with Nelson Day (1921a, 1921b), lieutenant of Royal Naval Reserve, and made at Fern-down, Dorset, England, was published by Markwick (1921), based on their correspondence within weeks of the sighting. This report conveys that, with others, Nelson Day saw, in the evening of August 7, a bright object, allegedly of magnitude  $-2$ , at an angular distance of  $4^\circ$  from the Sun, whose elevation above the horizon was about  $8^\circ$  and bearing approximately  $S45^\circ W$  relative to the object, referring presumably, as in ground navigation, to the horizontal coordinate system (Section 4.3). Although the time of observation was not stated, Markwick estimated it at 19:00 UT. We calculate that at Fern-down the Sun was  $8^\circ$  above the horizon at 18:44 UT and the sunset (the time of last contact) occurred at 19:44 UT. Markwick judged this account credible and we tend to concur, with the exception of the brightness that must be grossly underestimated. In Section 4.2 we argue that an object of apparent magnitude  $-2$  cannot be detected in daylight with the naked eye  $4^\circ$  from the Sun.

Another report, by Emmert (1921a, 1921b), described a daylight observation from Detroit, MI, on August 6 at 22:50 UT: the object was found to be  $5^\circ$  east of the Sun in azimuth and within  $0.5^\circ$  at the same elevation above the horizon as the Sun, for which Emmert gave an azimuth of  $90^\circ$  and an elevation of  $15^\circ$ . We compute that at the given time of observation, the Sun's azimuth was  $94.8^\circ$  and its elevation  $19.9^\circ$ . The accuracy of Emmert's estimates thus cannot compete with Campbell's, and his account does not offer credible information on the object's absolute position, a conclusion that is supported by Pearce (1921). At best, Emmert's observation offers a very approximate estimate of the object's angular distance from the Sun more than one day before Campbell et al.'s observation. It should be remarked that the publication of Emmert's (1921a) original report was dated as late as 1921 October 14 and that it obviously was not available in time for publication in the preceding issue, dated September 13, still more than 5 weeks after the event. It appears that Emmert tried to reconstruct his observation from memory long after he made it, an effort that is notoriously unreliable. By contrast, even though communicated via a third person (thus involving extra delays in mailing), the original letter of Nelson Day's (1921a) observation made a September 8 issue of the journal. Their very unequal weights notwithstanding, Campbell's, Nelson Day's, and Emmert's accounts offer a fairly consistent scenario for the object's sunward apparent motion over a period of more than 28 hours.

Generally supportive, but of a still lesser weight, was another report from England, by Fellows (1921a, 1921b, 1921c). Observing on August 7 from Wolverhampton, in the West Midlands, he detected with his binoculars a bright object, elongated in the direction of the Sun and of a distinct reddish hue, at about 20:30 UT, more than 30 minutes after sunset (for which we compute 19:53 UT as the time of last contact). Fellows remarked that the object, seen by him for only a few minutes, was very low above the horizon and he judged it to be about  $6^\circ$  from the Sun and a little south of it. Because the Sun had already set, the reported angular distance cannot be more than an extremely crude guess and hardly of any diagnostic value.

Further reports appear to be even less relevant. Observations of transient luminous bands at the Königstuhl and Sonneberg Observatories and elsewhere (Wolf 1921a; Hoffmeister 1921) after midnight UT from August 8 to 9, although interpreted by some in the literature as the Lick object's tail, must have been unrelated, because such phenomena were also observed during the night of August 5/6 (Wolf 1921a). Hoffmeister (1921) noted that the bands resembled auroral streamers. Wolf (1921b) was more inclined to a possible correlation only because he suspected that the Lick object may also have been observed in daylight at Plauen, Germany, on August 7.816 UT (at a distance of  $27^\circ$  away from the Lick position), in which case it would have to have been very close to the Earth. Since that object was different (and in all probability identical with Jupiter; Pearce 1921), an association between the Lick object and the luminous bands can safely be ruled out.

An observation of similar phenomena was reported by Kanda (1922). Between 9:35 and 9:55 UT on August 9 he noticed a band-like object  $2^\circ$  broad extending from about R.A. =  $9^h 48^m$ , Dec. =  $+14^\circ$  to R.A. =  $10^h 42^m$ , Dec. =  $+30^\circ$ . He was observing from Simo-Sibuya, near Tokyo, Japan, and speculated that the band could be a tail of the Lick object. If this feature was located outside the Earth's atmosphere, it would have hardly escaped attention of Campbell (1921d), who under superior observing conditions at Lick saw nothing at sunrise on August 9 (the first contact at 13:11 UT), about  $3\frac{1}{2}$  hr after Kanda's reported sighting. Several additional reports could under no circumstances refer to the Lick object and there is no point in listing them here.

#### 4. SEARCHING FOR POTENTIAL CANDIDATES OF A GENETICALLY RELATED COMET

The fundamental idea in our quest for determining the identity and origin of the 1921 Lick object is to search for a long-period comet, with which the object could share the common orbit and thereby be genetically associated. The task is to develop an approach strategy based on what we learned about the comet pairs and groups in the preceding sections of this paper in order to streamline and optimize the search.

A necessary condition for a genetically related comet is that its motion, adjusted for a time of perihelion passage, should emulate the Lick object's motion and therefore closely match its observed or estimated position(s). The issue is the degree of uncertainty that is introduced by this orbital emulation, because the errors involved define the degree of tolerance that is to be maintained in judging each candidate's chances.

An estimate of this uncertainty is illustrated on the known pair and trio of the genetically related comets investigated, respectively, in Sections 2.1 and 2.2. It makes no difference whether the incurred errors are established by emulating a companion's orbit with a primary's orbit, or vice versa. Choosing the first option, we selected an arbitrary time near perihelion of the companion and calculated its equatorial coordinates from its own set of orbital elements. Next, we tried to match this artificial position with the primary's orbit, using the companion's optimized perihelion time. This procedure required the introduction of a correction  $\Delta t_\pi$  to minimize the total residual,  $\Delta\Pi$ , consisting of the residuals in right ascen-

**Table 6**

ERRORS IN COMPANION'S PERIHELION TIME AND EQUATORIAL COORDINATES AND POSITION INTRODUCED BY EMULATING COMPANION'S MOTION WITH PRIMARY'S MOTION.

Fragment		Errors introduced by primary's orbit			
primary	companion	$\Delta t_\pi$ (days)	$\Delta R.A.$	$\Delta Dec.$	$\Delta \Pi$
C/1988 F1	C/1988 J1	+0.004	+0.2	0.0	0.2
C/1988 A1	C/1996 Q1	-0.059	-5.8	+5.1	7.7
C/1988 A1	C/2015 F3	+0.013	+6.5	+5.1	8.3

sion,  $\Delta R.A.$ , and declination,  $\Delta Dec.$  This exercise provided us with estimates of the errors that ought to be tolerated when an effort is made to emulate the companion's motion with the motion of the primary.

The results, in Table 6, indicate that for the pair of C/1988 F1 and C/1988 J1, whose orbits in Table 1 are seen to differ hardly at all (as a result of a relatively recent separation of the fragments; Section 2.1.1), the astrometric error is well below  $1'$  and the optimized perihelion time agrees with the companion's to better than 0.01 day. For the trio of the primary C/1988 A1 and the companions C/1996 Q1 and C/2015 F3, whose orbital elements in Table 4 differ often in the third significant digit (due to the fragments' separation during their parent comet's previous return to the Sun; Section 2.2.1), the astrometric errors are near  $8'$  and the perihelion times differ by up to several units of 0.01 day. This appears to be the order of errors that are introduced in a comet pair or group by emulating a member's motion by the motion of another member. We adopt accordingly that uncertainties in the astrometric positions of the Lick object that ought to be tolerated in our search for its genetically related comet are up to several arcminutes.

#### 4.1. Constraints Based on Campbell et al.'s Account

An important point is raised by Campbell's (1921d) statement that the Lick object was brighter than Venus in the same position and circumstances. Since any small elongation,  $E$ , from the Sun is reached by Venus either near the superior or inferior conjunction, we consulted the phase function of the planet to derive its apparent brightness for those two scenarios. With  $E = 3^\circ.3$  and the Earth's distance from the Sun of  $r_\oplus = 1.01387$  AU at the time, Venus' phase angle at its mean heliocentric distance would have been  $4^\circ.7$  near the superior conjunction and  $175^\circ.3$  near the inferior conjunction.

A history of the planet's phase function investigations from a standpoint of *The Astronomical Almanac's* needs was summarized by Hilton (2005). A recently updated phase curve, the only one that accounts for forward-scattering effects of sulfuric-acid droplets in the planet's upper atmosphere and covers all angles from  $2^\circ$  to  $179^\circ$ , was published by Mallama et al. (2006). Based on their results we find that at  $E = 3^\circ.3$  the  $V$  magnitude of Venus is  $-3.9$  and  $-3.7$  near the superior and inferior conjunction, respectively. Incorporating a minor color correction to convert the  $V$  magnitude to the visual magnitude (e.g., Howarth & Bailey 1980) and interpreting Campbell et al.'s statement to imply that the Lick object was at least  $\sim 0.5$  magnitude brighter than Venus would have been, we adopted that the object's apparent visual magnitude was about  $H_{\text{app}} = -4.3$  or possibly brighter.

Campbell et al.'s remarks on the Lick object allow us to examine the constraints on its magnitude  $H_0$ , normalized to unit heliocentric and geocentric distances (sometimes called an *absolute magnitude*), as a function of the unknown heliocentric distance  $r$  (in AU), on the assumption that the object's brightness varied with heliocentric distance as  $r^{-n}$ ,

$$H_0 = -4.3 - 5 \log \Delta - 2.5 n \log r - 2.5 \log \Phi(\varphi), \quad (27)$$

where  $n$  is a constant and  $\Delta$ ,  $\varphi$ , and  $\Phi(\varphi)$  are, respectively, the object's geocentric distance (in AU), phase angle, and phase function. The distances  $r$  and  $\Delta$  are constrained by the observed elongation angle  $E$ ,

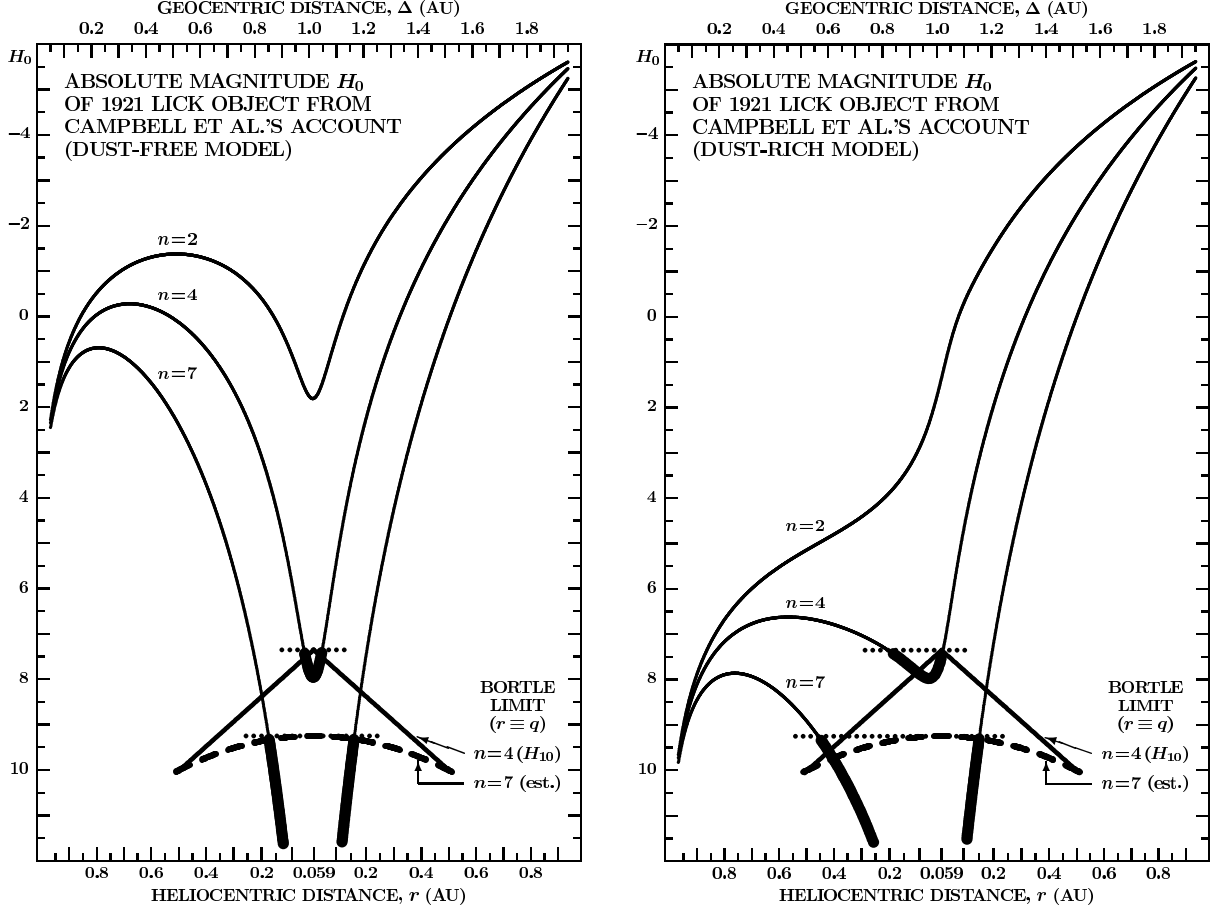
$$r^2 = r_\oplus^2 + \Delta^2 - 2r_\oplus\Delta \cos E, \quad (28)$$

where  $R_\oplus$  is again the earth's heliocentric distance at the time of Campbell et al.'s observation. Because the phase function depends on the object's unknown dust content and the small elongation implies strong effects of forward scattering by microscopic dust at the object's geocentric distances  $\Delta \lesssim 0.95$  AU, we constrained the phase function by employing two extreme scenarios: dust-free and dust-rich. In the first case we put  $\Phi(\varphi) \equiv 1$  at all phase angles, while in the second case we approximated  $\Phi(\varphi)$  with the Henyey-Greenstein law as modified by Marcus (2007) but normalized to a zero phase angle [ $\Phi(0^\circ) = 1$ ] rather than to  $90^\circ$ .

We varied  $\Delta$  from 0.001 AU to 2 AU to obtain a plot of the absolute magnitude  $H_0$  against  $r$  in Figure 5 for three values of the exponent  $n$ ; a dust-free case is on the left, a dust-rich case on the right. Although the models differ from one another enormously at  $\Delta < 1$  AU, some properties of the plotted curves are independent, or nearly independent, of the phase law.

Rather odd-looking in Figure 5 are the magnitude curves for  $n = 2$ , typical of dynamically new comets arriving from the Oort Cloud (Whipple 1978), which generally imply an unacceptably high intrinsic brightness of the Lick object. In the dust-free scenario the solutions are way off, while in the dust-rich model the acceptable options imply a very small geocentric distance  $\Delta < 0.2$  AU. But such scenarios are ruled out by (i) being inconsistent with Campbell et al.'s observation of the object's stellar appearance; (ii) failing to explain why the object was not observed systematically for a longer period of time around August 7 (at an expected rate of motion of up to several degrees per day); and (iii) explicitly contradicting Campbell's failure to find it on the following days at the times of sunrise and/or sunset. It is therefore highly unlikely that the Lick object was a dynamically new comet and it was not near the Earth.

Campbell's (1921d) unsuccessful search for the object on August 8–9 (Mount Hamilton time) suggests that by then it may have disintegrated. If it was a member of a comet pair and *not* a dynamically new comet, it should, like the members of the C/1988 A1 group, belong to Whipple's (1978) class III (with an orbital period on the order of thousands of years), for which Whipple found an average  $\langle n \rangle = 4.7 \pm 0.8$  before perihelion. Not unexpectedly, the preperihelion light curves of C/1988 A1 and C/1996 Q1 (up to the onset of terminal fading) in Figure 3 show that they, too, are fairly consistent with  $n = 4$  or slightly higher. The magnitude curves for  $n = 4$



**Figure 5.** Absolute magnitude  $H_0$  of the 1921 Lick object as a function of its assumed heliocentric distance at the time of Campbell et al.'s observation of sunset on Mount Hamilton on August 8.1347 UT. The  $H_0$  variations are plotted for three different photometric laws  $r^{-n}$ , with  $n = 2, 4$ , and  $7$  and for two dust-content scenarios: dust-free (the panel on the left) and dust-rich. The enormous divergence that the two display for geocentric distances smaller than 1 AU is due to strong effects of forward scattering by microscopic dust at large phase angles implied by the object's small elongation. Also plotted for  $n = 4$  is Bortle's (1991) survival limit  $H_{10} \equiv H_0(n=4)$  for comets with perihelion distances  $q \leq 0.51$  AU, when the plotted  $r$  is equal to  $q$ . For  $r > q$  the  $H_{10}$  value of surviving comets lies along a horizontal (examples for  $q = 0.059$  AU shown by dots) that passes through the Bortle curve at  $r = q$ . For  $n = 7$  we estimated the survival limit (dashed curve) on certain assumptions (see the text). The heavy portions of the magnitude curves for  $n = 4$  and  $n = 7$  show the ranges of heliocentric distance within which the object is unlikely to have survived. Statistically, about 70% of comets with the intrinsic magnitude brighter than the Bortle limit survive, and vice versa. The magnitude limit for  $q = 0$  is 7 for  $n = 4$  and it is estimated at 9.3 for  $n = 7$ .

in Figure 5 are thus expected to approximate class III comets more closely than the other curves. The possibility that the Lick object disintegrated near perihelion, just as did C/1996 Q1, suggests that it could be a companion to the pair's more massive member, which was likely to have arrived at perihelion long *before* the Lick object. Regardless of the dust content, Figure 5 shows that the minimum on the magnitude curves for  $n = 4$  is  $(H_0)_{\min} \simeq 8$ . Accordingly, the Lick object should have had a small perihelion distance,  $q \lesssim 0.1$  AU, and the small elongation reflected its true proximity to the Sun. Since the minimum absolute magnitude varies with  $n$  approximately as  $(H_0)_{\min} = 3.09n - 4.3$ , it equals  $\sim 10$  for  $n = 4.7$ . We remark that C/1996 Q1 had  $H_0 \simeq 6.6$  before fading (Figure 3), being intrinsically about 0.7 magnitude fainter than C/1998 A1 at  $\sim 1$  AU from the Sun.

The curve for  $n = 7$  is presented in Figure 5 as an example of an unusually steep light curve — displayed, e.g., by the sungrazing comet C/2011 W3 (Sekanina & Chodas 2012) — and to illustrate a degree of nonlinearity in the plot of the Lick object's  $H_0$  magnitude. Figure 3

shows that the  $r^{-7}$  law is not consistent with the preperihelion light curve of either C/1988 A1 or C/1996 Q1 and that it could be a plausible match only to the post-perihelion light curve of C/2015 F3.

To learn more about the Lick object's possible disintegration, we recall that a failure of long-period comets to survive perihelion was investigated, among others, by Bortle (1991). For such comets with perihelion distances smaller than 0.51 AU he found that the chance of disintegration depends on the intrinsic brightness. By fitting the same  $r^{-4}$  law to the light curves of the numerous comets that he examined, Bortle estimated that there was a 70% probability of disintegration for the comets that were intrinsically fainter than the absolute magnitude  $H_{10}^{\text{surv}}$  (equivalent in our notation to  $H_0$  for  $n = 4$  and  $\Phi = 1$ ). This survival limit is according to Bortle related to the perihelion distance  $q$  (in AU) by  $H_{10}^{\text{surv}} = 7 + 6q$ . It is plotted in Figure 5 against  $r$ , thus assuming the equality between the plotted  $r$  and the perihelion distance. For  $r > q$ ,  $H_{10}$  lies along a horizontal line that passes through the Bortle curve at  $r = q$ . The

Bortle limit suggests that comets intrinsically brighter than  $H_{10} = 7$  have an increasingly higher probability to survive, even if their perihelia are within, say, 0.1 AU of the Sun.

On certain assumptions, the Bortle limit can readily be extended to photometric laws  $r^{-n}$  with  $n \neq 4$ . For example, if an orbital arc covered by the magnitude observations, employed to determine the absolute magnitude, is centered on a heliocentric distance  $r_{\text{aver}}$  for which  $\log r_{\text{aver}} = \log(q + 0.5)$ , the survival limit  $H_{2.5n}^{\text{surv}}$  is related to  $H_{10}^{\text{surv}}$  by  $H_{2.5n}^{\text{surv}} = H_{10}^{\text{surv}} + (10 - 2.5n) \log(q + 0.5)$ . In Figure 5 we display, as a dashed curve, a survival limit  $H_{17.5}^{\text{surv}}$  for the magnitude curve of  $n = 7$ . For perihelion distances smaller than 0.5 AU,  $H_{17.5}^{\text{surv}}$  is fainter than the  $H_{10}^{\text{surv}}$  and its variation with the perihelion distance is much flatter.

While the minimum on the magnitude curve provides a lower bound to the Lick object's absolute brightness, the Bortle limit offers a sort of an upper bound, if the object's disintegration is considered as very likely. Figure 5 shows that for  $n = 4$  the two bounds almost coincide, leaving a narrow allowed range from 8.1 to 7.3. In the case of a dust-poor comet, such as was C/1996 Q1 (Section 2.2.2), the magnitude curve for  $n = 4$  in Figure 5 suggests that the Lick object's perihelion distance did not exceed 0.07 AU, while the same curve in the case of a dust-rich comet allows a perihelion distance of up to 0.20 AU. For the unlikely magnitude curves with  $n = 7$  the limits on the perihelion distance would be, respectively, 0.17 AU and 0.46 AU.

Given that the above bounds are not absolute, a search for the Lick object's pair member should allow for some leeway. However, it ought to be pointed out that if the apparent brightness assigned to the object, based on Campbell's comparison to Venus, should be increased to, say, magnitude  $-4.5$  or even  $-5$ , the curves in Figure 5 would move up and further reduce a range of conforming perihelion distances, compensating for any intrinsic brightness increase caused by softening the Bortle survival limit's constraint.

There is circumstantial evidence, to be discussed below, that the Lick object's brightness, as observed by Campbell et al. at sunset on August 7, was a result of more complex temporal variations than a simple power law of heliocentric distance can explain. If this indeed was so, our conclusion that, based on Campbell et al.'s account, the object was truly in close proximity of the Sun (and not only in projection onto the plane of the sky) should further be strengthened.

#### 4.2. Daytime Observations

The Lick object's observations by Nelson Day (Markwirk 1921), on August 7, 8.5 hr before Campbell et al.'s sighting, by Emmert (1921a, 1921b), on August 6, another 20 hr earlier, and possibly by others, were made with the naked eye in broad daylight, with the Sun a number of degrees above the horizon. This kind of visual observation has both advantages and disadvantages over twilight observations, such as that made by Fellows (1921a, 1921b). An obvious advantage is that the angular distance of the object from the Sun is readily observed. A disadvantage that has implications for the Lick object's physical behavior is a potentially enormous uncertainty in the estimated brightness.

In Section 3 we already hinted that Nelson Day's report of the Lick object's apparent magnitude of  $-2$  at  $4^\circ$  from the Sun on August 7 must be grossly underrated. The issue of daylight visual-magnitude estimates of bright celestial objects was addressed by Bortle (1985, 1997), who presented a formula for a limiting magnitude of an object just detectable with 8-cm binoculars as a function of its elongation from the Sun. At  $4^\circ$  from the Sun one could observe with this instrument an object as faint as  $-1.3$ . Furthermore, on a website Bortle posted a list of threshold visual magnitudes for daytime observations with the unaided eye.<sup>10</sup> From this document it follows, for example, that an object of magnitude  $-4.0$  is just visible with the unaided eye if more than  $5^\circ$  from the Sun, while an object of  $-5.5$  or somewhat brighter is rather easily visible when the Sun is blocked. Since, in reference to the Ferndown observation, Markwirk (1921) conveyed that the Lick object looked "striking," it must have been at this time quite a bit brighter than magnitude  $-4$ , possibly even brighter than  $-5$ .

Essentially the same constraint applies to Emmert's sighting on August 6. Emmert estimated the Lick object's brightness by asserting that it "was fully as bright as Venus in twilight at her greatest brilliancy" (Emmert 1921a),<sup>11</sup> while elsewhere he claimed that it "shone altogether too bright for Venus" (Emmert 1921b). The two statements are not incompatible, as the latter refers implicitly to the planet's expected brightness (fainter than  $-4$ ; Section 4.1) at the time of observation; the former statement implies a magnitude just shy of  $-5$ .

We cannot rule out that the Lick object was getting fainter as it was approaching the Sun, which brings to mind the instances of post-outburst terminal fading of disintegrating comets, including C/1996 Q1 (Figure 3). It is possible that both Emmert and Nelson Day saw the Lick object during its final outburst, while Campbell's (1921d) observation was made along the outburst's subsiding branch and his unsuccessful search on the following days after the event was over.

#### 4.3. Astrometric Data and Other Tests in Search for a Genetically Related Comet

We were now ready to undertake a search for a comet (or comets), whose orbit could serve as a basis for fitting the limited data on the Lick object, as described in Section 3. Performed in Equinox J2000, the search rested on the following set of diagnostic tests of the evidence:

(1) The most credible data were provided by Campbell (1921d): the object's position for 1921 August 8.1347 UT, R.A.(J2000) =  $9^{\text{h}}27^{\text{m}}.5$ , Dec.(J2000) =  $+15^\circ04'$ , whose uncertainty is estimated at  $\pm 10'$ .

(2) The observation by Nelson Day on 1921 August 7.7806 UT (Markwirk 1921), placing the object  $4^\circ$  from the Sun, was judged less reliable than Campbell's and was employed as a second, less diagnostic test, with an estimated uncertainty of  $\pm 30'$ . The somewhat cryptic depiction of the object-Sun orientation (Section 3) suggests,

<sup>10</sup> J. E. Bortle's one page document, titled "Judging Potential Visibility of Daylight Comets," is available from a website <http://www.eagleseye.me.uk/DaylightComets.pdf>.

<sup>11</sup> From Mallama et al.'s (2006) phase curve we find that at Earth's and Venus' mean heliocentric distances, the peak apparent magnitude of Venus is  $V = -4.81$  at a phase angle of  $123^\circ.55$  and elongation of  $37^\circ.07$ .

if correctly interpreted, the object's equatorial coordinates of R.A.(J2000) =  $9^{\text{h}}29^{\text{m}}.6$ , Dec.(J2000) =  $+16^{\circ}29'$ , with an estimated uncertainty of more than  $\pm 30'$ .

(3) The observation by Emmert (1921a, 1921b) was judged to furnish information that was much less credible than Campbell's and less credible than Nelson Day's. We employed the object's reported angular distance from the Sun,  $5^{\circ}$  on 1921 August 6.9514 UT, as the only piece of data worth testing; its uncertainty was estimated at  $\pm 1^{\circ}$  at best.

To explain the Lick object's sudden appearance, disappearance, and other circumstances, it was judged helpful if a genetically related comet and its orbit satisfied four additional constraints:

(4) The most likely reason for the object's sudden appearance was thought to be its unfavorable trajectory in the sky during its long approach to perihelion: from behind the Sun, along a narrow orbit that would entail a small elongation from the Sun over an extended period of many weeks before near-perihelion discovery.

(5) A very small perihelion distance,  $q < 0.1$  AU, was preferred because it would most straightforwardly account for the object's brief brightening, thus aiding the argument in the previous point. It would also expose the object to a highly perilous environment near perihelion, thus increasing the chances of its disintegration and thereby its abrupt disappearance (Section 4.1).

(6) Since in comet pairs and groups the chance of near-perihelion demise was limited to companions (Section 2), which typically arrived at perihelion much *later* than the primary members, one would expect that the Lick object was a companion to the pair's primary and, accordingly, that the primary would more probably have arrived at perihelion well *before* 1921.

(7) Expressed by Equation (2), the correlation between (i) a velocity at which the fragments begin to separate after their parent's breakup near perihelion and (ii) a temporal gap between their observed arrivals to perihelion one revolution later, favors, for a typical gap of dozens of years, fragments of a comet with an orbital period on the order of thousands of years. Thus, such a comet is more likely to be genetically related to the Lick object than a comet with a shorter or longer orbital period; we address the preferred range for our test purposes in Section 5.

Having described the tests and constraints that the comets potentially associated with the Lick object were expected to satisfy, we next compiled an initial list of selected candidates. We allowed a wide range of orbital periods,  $P$ , by letting the statistically averaged separation velocity,  $\langle V_{\text{sep}} \rangle$  [for the definition, see the text below Equation (26)] to vary from 0.1 to 3 m s $^{-1}$ ; the difference in the orbital period,  $\Delta P$ , between the primary and the companion, to range from 3 to 300 yr; and the heliocentric distance at fragmentation,  $r_{\text{frag}}$ , to span 0.01 to 10 AU. Expressing  $\Delta V$  from Equation (2) in terms of  $\langle V_{\text{sep}} \rangle$ , the orbital periods range from 100 to 100 000 yr. Requiring conservatively that the perihelion distance not exceed  $\sim 0.25$  AU, we assembled all comets from Marsden & Williams' (2008) catalog that satisfied these conditions (the orbital period as derived from the original semimajor axis; for C/1769 P1 computed from an osculating value by R. Kracht), and with orbits of adequate precision (a minimum of five decimals in  $q$  and three in the

**Table 7**  
POTENTIAL CANDIDATES FOR A COMET THAT COULD BE  
GENETICALLY RELATED TO THE LICK OBJECT.

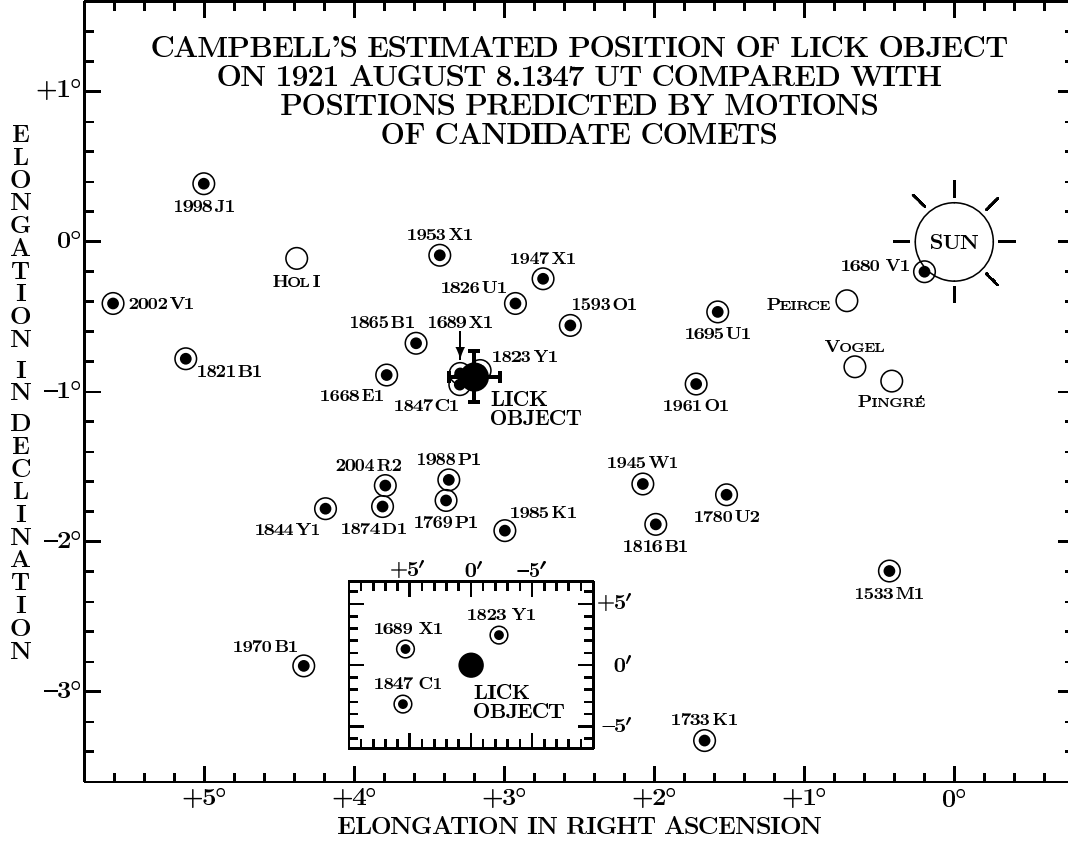
Candidate comet	Perihelion distance, $q$ (AU)	Orbital period, <sup>a</sup> $P_{\text{orig}}$ (yr)	Inclination, $i$
C/1533 M1	0.2548	.....	149°59
C/1577 V1	0.1775	.....	104.85
C/1582 J1	0.1687	.....	118.54
C/1593 O1	0.0891	.....	87.91
C/1665 F1	0.1065	.....	103.89
C/1668 E1	0.0666	.....	144.38
C/1680 V1	0.0062	9530	60.68
C/1689 X1	0.0644	.....	63.20
C/1695 U1	0.0423	.....	93.59
C/1733 K1	0.1030	.....	23.79
C/1737 C1	0.2228	.....	18.33
C/1743 X1	0.2222	.....	47.14
C/1758 K1	0.2154	.....	68.30
C/1769 P1	0.1228	2100*	40.73
C/1780 U2	0.0993	.....	126.18
C/1816 B1	0.0485	.....	43.11
C/1821 B1	0.0918	.....	106.46
C/1823 Y1	0.2267	.....	103.82
C/1826 U1	0.0269	.....	90.62
C/1827 P1	0.1378	.....	125.88
C/1831 A1	0.1259	.....	135.26
C/1844 Y1	0.2505	7590	45.57
C/1847 C1	0.0426	8310	48.66
C/1851 U1	0.1421	.....	73.99
C/1859 G1	0.2010	.....	95.49
C/1865 B1	0.0258	.....	92.49
C/1874 D1	0.0446	.....	58.89
C/1901 G1	0.2448	.....	131.08
C/1905 X1	0.1902	.....	43.65
C/1917 F1	0.1902	143	32.68
C/1927 X1	0.1762	14 600	84.11
C/1945 W1	0.1944	.....	49.48
C/1947 X1	0.1100	3720	138.54
C/1948 L1	0.2076	83 100	23.15
C/1948 V1	0.1354	21 500	23.12
C/1953 X1	0.0723	.....	13.57
C/1959 Q2	0.1655	.....	48.26
C/1961 O1	0.0402	44 900	24.21
C/1967 M1	0.1783	.....	56.71
C/1970 B1	0.0657	.....	100.18
C/1975 V1	0.1966	16 300	43.07
C/1985 K1	0.1063	.....	16.28
C/1987 W1	0.1995	.....	41.62
C/1988 P1	0.1646	.....	40.20
C/1996 B2	0.2302	16 400	124.92
C/1998 J1	0.1532	.....	62.93
C/2002 V1	0.0993	9080	81.71
C/2004 F4	0.1693	2690	63.16
C/2004 R2	0.1128	.....	63.17
C/2004 V13	0.1809	.....	34.81

<sup>a</sup> The asterisk indicates the original orbit was derived by the second author; parabolic orbits are shown by dots.

angular elements). For the period of 2008–2015 we consulted the JPL Small-Body Database Search Engine.<sup>12</sup>

The candidate comets are presented in Table 7. In the Marsden-Williams catalog we found merely 14 ob-

<sup>12</sup> See <http://ssd.jpl.nasa.gov/sbdb-query.cgi>.



**Figure 6.** Position of the Lick object relative to the Sun on 1921 August 8.1347 UT (solid circle), reported by Campbell (1921d), and its predicted positions (circled dots), derived by emulating the object’s motion with the motions of potentially related comets, listed in Table 7. The plot contains all 26 comets (of the 50) that fit its limits. Three comets — C/1823 Y1, C/1689 X1, and C/1847 C1 (Hind) — match the Lick object’s position to within  $10'$ . For C/1689 X1 we show not only the result from the cataloged orbit (circled dot), but also from the other available orbits (open circles), marked by the name of the computer (see also Tables 10 and 11). The size of the Sun is drawn to scale. On a scale expanded by a factor of nearly five, the inset displays the three comets, whose orbits match the position of the Lick object most closely. The width of the inset window equals the estimated uncertainty of the Lick object’s position.

jects that satisfied our selection constraints, including C/1917 F1 (Mellish), a parent body of an extensive meteor-stream complex (e.g., Neslušan & Hajduková 2014). We found no candidate comets from 2005 to 2015. Since comets with an orbital period between 100 and 100 000 yr may easily appear among those for which only a parabolic approximation is available, we extended the set of candidates by adding small-perihelion comets with parabolic orbits. Even though we argued that the comets before 1921 should be the more attractive candidates [see the point (6) above], we allowed the comets that arrived after 1921 to be subjected to the same tests. We ended up with a total of 50 candidate comets.

##### 5. RESULTS OF A SEARCH FOR A COMET POTENTIALLY RELATED TO THE LICK OBJECT

Since we assigned the greatest weight to the position of the Lick object on August 8.1347 UT, as reported by Campbell (1921d), test 1 was applied first. The object’s position relative to the Sun is in Figure 6 compared with the positions predicted by the orbits of 26 of the 50 candidate comets listed in Table 7. The remaining 24 lie outside the plot’s limits. There are three comets whose cataloged orbits match the Lick object’s position estimated by Campbell highly satisfactorily, well within  $10'$ , its expected uncertainty (Section 4.3). In chronological

order, the three objects are C/1689 X1, C/1823 Y1, and C/1847 C1 (Hind), all of them great, intrinsically bright comets. Based on this evidence alone, the Lick object could be genetically related to any one of the three. We note that they all preceded the arrival of the Lick object, as preferred by test 6.

Table 8 presents the ten top candidates for a comet genetically related to the Lick object in the order of increas-

**Table 8**  
LIST OF 10 COMETS WITH ORBITS THAT MOST CLOSELY MATCH CAMPBELL’S POSITION FOR THE LICK OBJECT.

Candidate comet	Offset		Time diff. $t_{\text{obs}} - t_{\pi}$ (days)	Distance (AU) to		Phase angle
	Dist.	P.A.		Sun	Earth	
C/1823 Y1	3.4	317°	-2.266	0.2410	0.7784	166°0
C/1689 X1	5.5	76	-1.285	0.1050	0.9263	144.8
C/1847 C1	6.4	119	-0.667	0.0682	1.0426	63.4
C/1865 B1	26.9	60	-0.713	0.0713	0.9821	114.6
C/1826 U1	33.6	331	+0.479	0.0530	1.0049	98.2
C/1668 E1	34.1	89	+1.021	0.0933	1.0743	47.7
C/1988 P1	42.5	166	+0.261	0.1649	1.1626	23.8
C/1593 O1	43.5	298	-0.177	0.0897	1.0894	31.4
C/1947 X1	47.9	325	-0.652	0.1152	1.1170	25.1
C/1953 X1	50.5	16	-0.202	0.0736	1.0535	55.7



**Table 9**  
TESTS FOR CANDIDATE COMETS THAT COULD BE GENETICALLY RELATED TO THE LICK OBJECT.

Test No.	Lick object's test description	Condition (observation/desirability)	Potential genetically related comet		
			C/1689 X1	C/1823 Y1	C/1847 C1
1	Observation 1921 August 8.1347 UT:				
	Elongation <sup>a</sup>	3°.3	3°.4	3°.3	3°.4
	Elongation in R.A. <sup>a</sup>	+3°.2	+3°.3	+3°.2	+3°.3
	Elongation in Dec. <sup>a</sup>	-0°.9	-0°.9	-0°.9	-1°.0
2	Observation 1921 August 7.7806 UT:				
	Elongation <sup>b</sup>	~4°	4°.5	3°.2	4°.0
	Elongation in R.A. <sup>c</sup>	(+4°.0)	+3°.9	+2°.5	+4°.0
	Elongation in Dec. <sup>c</sup>	(+0°.4)	-2°.3	-2°.0	-0°.1
3	Observation 1921 August 6.9514 UT:				
	Elongation <sup>d</sup>	~5°	7°.4	4°.5	5°.3
4	Approach trajectory in the sky:				
	Maximum elongation at $r < 2$ AU	small enough to preclude early detection	85°	49°	46°
	Overall trend in elongation with time	should be decreasing	steadily decreasing	steadily decreasing <sup>e</sup>	steadily decreasing
	Approach direction in space	from behind the Sun	from behind the Sun	from behind the Sun	from behind the Sun
5	Perihelion distance	<0.1 AU	0.064 AU	0.227 AU	0.043 AU
6	Perihelion arrival relative to Lick object	years earlier to fit condition (2)	231.7 yr earlier	97.7 yr earlier	74.4 yr earlier
7	Orbital period (since previous return)	between ~2000 and ~50 000 yr	unknown	~2100 yr <sup>f</sup>	~8300 yr

<sup>a</sup> Estimated uncertainty of  $\pm 10'$ .

<sup>b</sup> Estimated uncertainty of  $\pm 30'$ .

<sup>c</sup> Condition subject to validity of employed interpretation of reported position (Sections 3 and 4.3); if valid, estimated uncertainty of more than  $\pm 30'$ .

<sup>d</sup> Estimated uncertainty of  $\pm 1^\circ$  at best.

<sup>e</sup> Except in close proximity of perihelion, starting about 2.5 days before the passage.

<sup>f</sup> See Section 5.2.

ing offset. Columns 2 and 3 list, respectively, the offset and the position angle from the Lick object's position estimated by Campbell (1921d); column 4 shows whether the observation, at time  $t_{\text{obs}}$ , was made before or after the candidate's perihelion,  $t_\pi$ ; columns 5 and 6 provide, respectively, the heliocentric and geocentric distances at  $t_{\text{obs}}$ , describing the role of geometry; and the last column lists the phase angle, which allows one to assess the effect of forward scattering on the object's observed brightness. Starting with the fourth candidate, C/1865 B1, the chances of a genetic relationship with the Lick object are highly unlikely, as from there on the offsets exceed the estimated uncertainty by more than a factor of two. We list these comets to accentuate the existence of a remarkable gap between the first three entries and the rest.

In the following we focus on the three promising candidates, whose comparison in terms of all seven tests formulated in Section 4.3 is offered in Table 9. Although the range of orbital periods of the candidate comets in Table 7 spanned three orders of magnitude, the orbital period of a likely candidate is now expected to be more tightly constrained, because the lower limit to the heliocentric distance at fragmentation and the temporal gap between the arrival times of the candidate comets and the Lick object are known. Test 7 in Table 9 reflects this tightening of the range of orbital periods, whose crude limits span a range from ~2000 to ~50 000 yr. We are now ready to inspect each of the three contenders separately in chronological order.

### 5.1. Comet C/1689 X1

Close examination of the history of this comet's orbit determination shows that the study by Holetschek (1891, 1892), whose "best" orbital set was cataloged by Marsden & Williams (2008) and used by us, was preceded by orbital investigations of Vogel (1852a, 1852b), B. Peirce (Kendall 1843), and Pingré (1784).<sup>13</sup> There are very dramatic differences among the computed orbits, which are traced to different interpretations of the crude observations. Although the comet was discovered about a week before perihelion (Kronk 1999) and was later observed also on the island of Ternate, Indonesia (Pingré 1784) and in China (Struyck 1740; Pingré 1784; Hasegawa & Nakano 2001), its approximate positions could only be constructed from the accounts reported on four mornings after perihelion by the observers at Pondicherry, then a French territory in India (Richaud 1729), and at Malacca, Malaysia (de Bèze & Comilh 1729).<sup>14</sup> The

<sup>13</sup> Converted to the equinox of J2000, the orbital sets from all four investigations are conveniently summarized by G. W. Kronk at a website [http://cometography.com/orbits\\_17th.html](http://cometography.com/orbits_17th.html).

<sup>14</sup> The name of de Bèze's co-worker was misspelled in the original French publication as *Comille*; no one by that name ever worked with de Bèze, yet the error propagated through the literature as it was copied over and over again for nearly three centuries [Kronk (1999) misspells both de Bèze and Comilh]. All three mentioned observers of C/1689 X1, Richaud, de Bèze, and Comilh were among 14 French Jesuits sent by Louis XIV as "royal mathematicians" to King of Siam to assist him in implementing his intention to found a scientific academy and an observatory (e.g., Udías 2003; Hsia 2009).

**Table 10**

SETS OF PARABOLIC ORBITAL ELEMENTS OF C/1689 X1  
DERIVED BY J. HOLETSCHEK (EQUINOX J2000).

Quantity/Orbital element	Catalog	HOL I	HOL II
Perihelion time $t_\pi$ (1689 Nov. UT)	30.659	30.545	28.461
Argument of perihelion $\omega$	78°.134	163°.857	143°.535
Longitude of ascending node $\Omega$	283°.754	67°.841	51°.682
Orbit inclination $i$	63°.204	143°.211	135°.308
Perihelion distance $q$	0.06443	0.09281	0.16673

difficulties encountered in an effort to determine this comet’s orbit were befittingly summarized by Plummer (1892), including the intricacies that Holetschek (1891, 1892) got involved with in order to accommodate a constraint based on a remark by de Bèze and Comillh that the comet reached a peak rate of more than 3° per day in apparent motion between 1689 December 14 and 15. Unable to find an orbital solution that would satisfy this constraint, Holetschek himself eventually questioned the report’s credibility, considering his preferred orbital solution (based on the adopted positions from December 10, 14, and 23 and on some experimentation with the observation time on the 10th) as a compromise.

Once the condition of a peak rate of apparent motion is dropped, there is no longer any reason to prefer this compromise solution over other available orbital sets. Accordingly, we examined whether or not the orbits by Pingré (1784), by Peirce (Kendall 1843; Cooper 1852), by Vogel (1852a, 1852b), and the alternative orbits by Holetschek (1891) himself matched Campbell’s position for the Lick object as closely as did the cataloged orbit for C/1689 X1. The two alternative orbits by Holetschek (1891) that we considered are compared to the cataloged orbit in Table 10. The first set of elements, referred to as “HOL I” in Table 10, was a key solution that Holetschek derived before he began a complicated data manipulation. The second set, referred to as “HOL II” in Table 10, is one of three considered by him — in the course of another effort to accommodate the high rate of apparent motion — to avoid the tendency of the computed path to have strayed to the northern hemisphere before perihelion, and the only of the three to have the perihelion distance smaller than 0.25 AU (thus complying with our selection rules). It is somewhat peculiar that Holetschek’s orbit in the Marsden-Williams catalog is the only set of elements (among the six we consider for C/1689 X1) that shows the comet’s motion to be direct; the rest are all retrograde, with the inclinations between 110° and 150°.

**Table 11**

ORBITS OF C/1689 X1 EMULATING CAMPBELL’S POSITION  
FOR THE LICK OBJECT.

Computer/ designation	Offset		Time diff. $t_{\text{obs}} - t_\pi$ (days)	Distance (AU) to		Phase angle
	Dist.	P.A.		Sun	Earth	
Catalog	5'.5	76°	−1.285	0.1050	0.9263	144°8
HOL I	85.4	56	−0.098	0.0930	0.9596	123.5
HOL II	371.6	50	−0.517	0.1682	0.9257	117.3
Vogel	152.3	272	+0.081	0.0211	1.0060	111.2
Peirce	152.1	282	+0.279	0.0386	1.0493	22.9
Pingré	167.0	270	+0.989	0.0948	1.1065	11.5

In Table 11, organized in the same fashion as Table 8, the various orbits for C/1689 X1 are compared in terms of the quality of matching the Lick object’s position published by Campbell. The table shows that due to enormous uncertainties in the orbit determination, the very small offset in Figure 6 and Table 8 from the cataloged set of elements must be judged as purely fortuitous. Accordingly, a genetic relationship between the Lick object and C/1689 X1 is on account of the orbital uncertainties entirely indeterminable. Arguments that tests 3 and 4 in Table 9 are unfavorable to the relationship are — while correct — under these circumstances unnecessary.

### 5.2. Comet C/1823 Y1

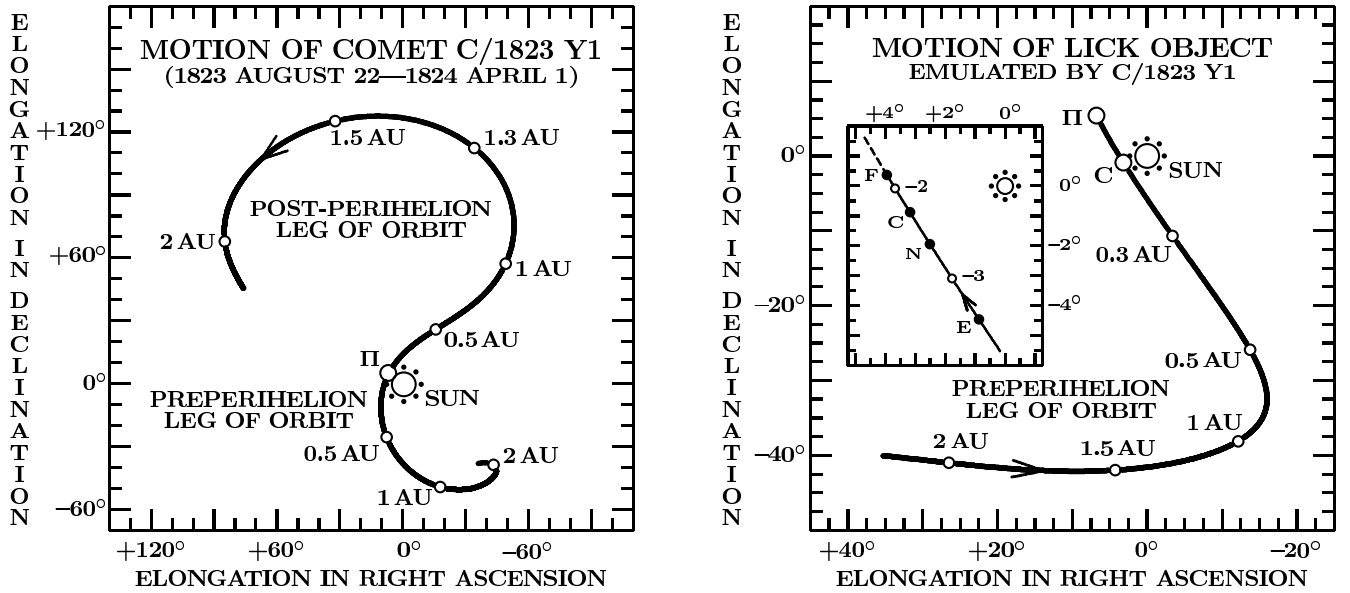
This was an intrinsically bright comet. On the assumption that its brightness, normalized to a unit geocentric distance and corrected for a phase effect according to the law by Marcus (2007), varied with heliocentric distance as  $r^{-n}$ , a least-squares solution to several magnitude estimates collected by Holetschek (1913) yields an absolute magnitude of  $H_0 = 3.70 \pm 0.22$  and  $n = 3.62 \pm 0.43$ . At first sight it is surprising that the comet was not discovered before perihelion, but a straightforward explanation is provided by Figure 7, which shows that the approach to perihelion was from the high southern declinations: the culprit was the lack of comet discoverers in the southern hemisphere in the early 19th century. For two months, from the beginning of 1823 September to the beginning of November, the comet was nearly stationary relative to the Sun, at elongations between 55° and 62° and at heliocentric distances from 2.2 AU to 1.1 AU.

The comet was poorly placed for observation in an early post-perihelion period as well, as it took three weeks before its elongation increased to 40°, at which time it was discovered as a naked-eye object<sup>15</sup> independently by several observers.

The most comprehensive orbit for this comet to date was computed by Hnatek (1912) from hundreds of astrometric observations made between 1823 December 30 and 1824 March 31. The perturbations by five planets, Venus through Saturn, were accounted for. An unusual feature of Hnatek’s study, marring its scientific value, was that while he computed both the most probable parabola and the most probable ellipse (with an osculation orbital period of 9764 yr) in the process of orbit refinement, he presented only a parabolic solution as the comet’s final orbit, dropping the last normal place (because of large residuals) and failing to demonstrate that the error of the eccentricity exceeded its deviation from unity.

Before deciding whether or not to redetermine the orbit with increased attention to its possible ellipticity, we noticed numerous accounts in the literature of the comet’s unusual appearance over a period from 1824 January 22 through 31, around the time of the Earth’s transit across the comet’s orbit plane on January 24.0 UT. At least five observers (Hansen 1824; Harding 1824; Olbers 1824b; von Biela 1824; Gambart 1825) reported that the comet displayed, next to its ordinary tail in the antisolar direction, a second extension that pointed sunward and was referred to by both Harding and Olbers as an *anomalous*

<sup>15</sup> According to Olbers (1824a), in the morning of 1824 January 5 the brightness of C/1823 Y1 equaled that of a star of magnitude 3; this was the comet’s first brightness estimate (Holetschek 1913).



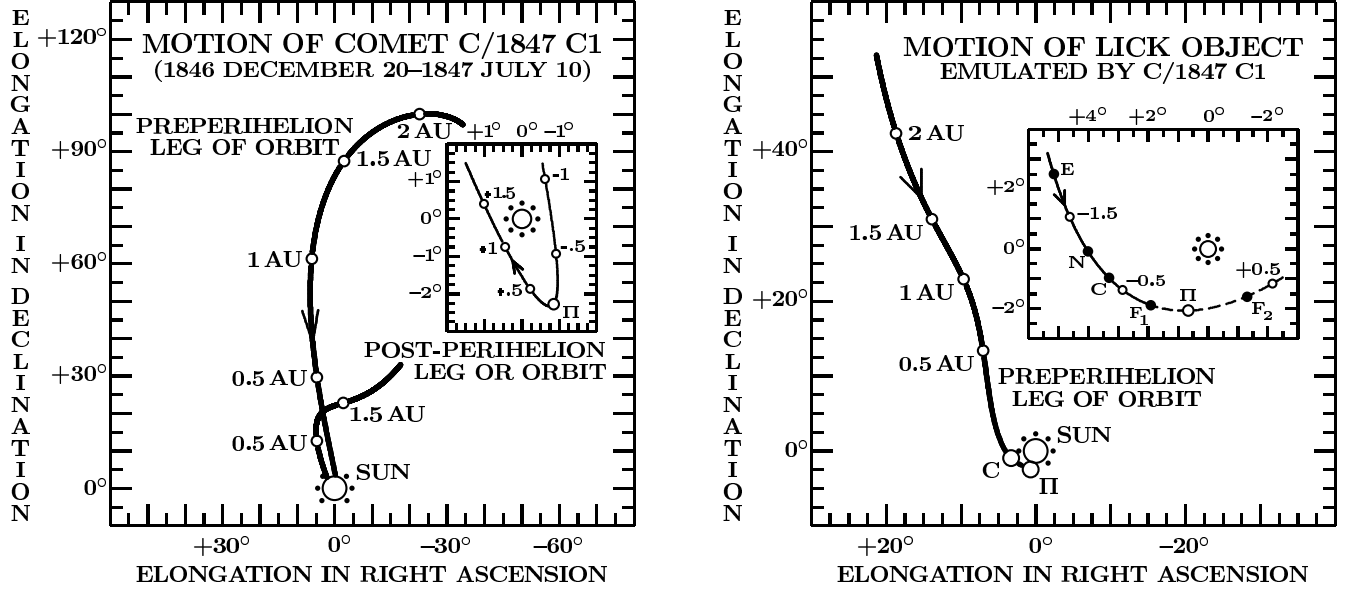
**Figure 7.** Apparent motions, relative to the Sun, of comet C/1823 Y1 (left) and of the Lick object, emulated by the comet's motion (right). The scale of the plot on the right is  $\sim 3.6$  times the scale on the left;  $\Pi$  is the perihelion point on, respectively, 1823 December 9.9 (left) and 1921 August 10.4 (right). Heliocentric distances at the comet's positions and the Lick object's predicted positions are depicted by the open circles. The elongation is reckoned negative to the south and the west of the Sun. — Left: The comet's motion from 1823 August 22 (110 days before perihelion) to 1824 April 1 (113 days after perihelion). The comet was almost stationary relative to the Sun for two months, from early September to early November. — Right: The Lick object's emulated motion from 1921 May 1 (101 days before perihelion) to perihelion; C is the object's predicted location at the time of Campbell et al.'s observation on 1921 August 8.135 UT. Closeup of the object's emulated motion near perihelion is shown in the inset, whose scale is 4 times the scale of the panel and in which the size of the Sun is drawn to the scale. The small open circles show the object's predicted positions 3 and 2 days before perihelion, respectively; C, N, and E refer to the object's locations at the times of Campbell et al.'s, Nelson Day's, and Emmert's observations, respectively; and F is the first of the three occasions, at sunrise on August 8 at Lick, on which Campbell failed to recover the object.

tail. Exhibited since by many other comets, most memorably by C/1956 R1 (Arend-Roland) and C/1973 E1 (Kohoutek), this type of comet-dust feature is nowadays customarily known as an *antitail* and its dynamics, governed by the conservation-of-orbital-momentum law, and appearance, controlled in large part by conditions of the projection onto the plane of the sky, are well understood (for a review, see Sekanina et al. 2001 and the references therein). The appearance of an antitail is clearly a signature of a comet rich in large-sized dust, but it is not diagnostic of its orbital category. The statistics show that both dynamically new comets, arriving from the Oort Cloud, and dynamically old comets (with the orbital periods on the order of thousands to tens of thousands of years) did show an antitail in the past. Indeed, as Marsden et al. (1973, 1978) showed, both C/1956 R1 and C/1973 E1 were dynamically new comets, as were several additional objects, e.g., C/1895 W1 (Perrine), C/1937 C1 (Whipple), C/1954 O1 (Vozárová), or, quite recently, C/2011 L4 (PanSTARRS); while other antitail-displaying objects, such as C/1844 Y1, C/1961 O1 (Wilson-Hubbard), C/1969 T1 (Tago-Sato-Kosaka), or, recently, C/2009 P1 (Garradd) and C/2014 Q2 (Lovejoy) were *not* dynamically new comets. As part of a broader antitail investigation, the feature in C/1823 Y1 was investigated by Sekanina (1976) and found to consist of submillimeter-sized and larger dust grains released from the nucleus before perihelion as far from the Sun as Jupiter, if not farther away still.

Given the orbit ambivalence in the antitail statistics, we decided to go ahead with the orbit redetermination, in order to ascertain whether C/1823 Y1 is in compliance

with test 7 of Table 9. Employing a code *EXORB7* developed by A. Vitagliano, the second author performed the computations, using selected consistent astrometric observations made between 1824 January 2 and March 31, which were re-reduced with the help of comparison-star positions from the *Hipparcos* and *Tycho 2* catalogs. The results are described in the Appendix; here we only remark that the original semimajor axis came out to be  $(1/a)_{\text{orig}} = +0.006077 \pm 0.000471 \text{ (AU)}^{-1}$ , implying a true orbital period of  $\sim 2100$  yr and demonstrating that the comet had not arrived from the Oort Cloud.

Having determined that C/1823 Y1 moved in an orbit that did not rule out its potential association with the Lick object (test 7), we next examined the comet's compliance with the other constraints summarized in Table 9. A parameter that is a little outside the proper range is the perihelion distance (test 5). The Lick object's motion along the orbit of C/1823 Y1 should have two implications for the observing geometry at the time of Campbell et al.'s observation, as seen from Table 8: (i) the object would have been only 0.78 AU from the Earth and (ii) its apparent brightness should have been strongly affected by forward scattering, if the object was as dust rich as C/1823 Y1. We reckon that for an  $r^{-4}$  law of variation, the Lick object would have appeared to Campbell et al.  $\sim 11.5$  mag brighter than was its absolute magnitude  $H_0$ . With an estimated apparent visual magnitude of  $-4.3$ ,  $H_0 \approx 7.2$ . With an  $r^{-5}$  law, the absolute brightness would drop to  $H_0 \approx 8.7$ . If the Lick object was caught by Campbell et al. in the midst of the terminal outburst's subsiding branch, its absolute brightness in quiescent phase could have been fainter still.



**Figure 8.** Apparent motions, relative to the Sun, of comet C/1847 C1 (left) and of the Lick object, emulated by the comet's motion (right). The scale of the plot on the right is twice the scale on the left;  $\Pi$  is the perihelion point on, respectively, 1847 March 30.8 (left) and 1921 August 8.8 (right). Heliocentric distances at the comet's positions and the Lick object's predicted positions are depicted by the open circles. The elongation is reckoned negative to the south and the west of the Sun. — Left: The comet's motion from 1846 December 20 (101 days before perihelion) to 1847 July 10 (101 days after perihelion). Unlike C/1823 Y1, C/1847 C1 was approaching the Sun from the north. Closeup of the comet's motion near perihelion is displayed in the inset, whose scale is 10 times the scale of the panel and in which the size of the Sun is drawn to the scale. The small open circles show the comet's positions 1 and 0.5 days before perihelion and 0.5, 1, and 1.5 days after perihelion. — Right: The Lick object's emulated motion from 1921 May 1 (100 days before perihelion) to perihelion; C refers to the location of the object at the time of Campbell et al.'s observation on 1921 August 8.135 UT. Closeup of the Lick object's emulated motion near perihelion is shown in the inset, whose scale is 4 times the scale of the panel and in which the size of the Sun is drawn to the scale. The small open circles show the object's predicted positions 1.5 and 0.5 days before perihelion and 0.5 days after perihelion; C, N, and E refer to the object's locations at the times of Campbell et al.'s, Nelson Day's, and Emmert's observations, respectively; and  $F_1$  and  $F_2$  are the first two occasions, at sunrise and sunset on August 8 at Lick, on which Campbell failed to recover the object.

While we find that the range of absolute magnitude is plausible for a companion of an intrinsically bright long-period comet, this model for the Lick object has difficulties that involve other tests besides test 5. The emulated trajectory for the period of August 6–8 predicts that the object should have moved from the southwest to the northeast (Figure 7), passing closest to the Sun, at an elongation of  $3^\circ.16$ , on August 7.867 UT, about 2 hours after Nelson Day's observation. As a result, the object's modeled elongation was increasing with time when Campbell et al. detected it. While this by itself is not contradicted by the observation (because the direction of the object's motion relative to the Sun during the several minutes of monitoring at Lick could not be established), the predicted elongation of  $3^\circ.2$  at the time of the Ferndown observation is not consistent with the observed  $4^\circ$  elongation and its estimated uncertainty of  $\pm 30'$  (test 2 in Table 9).

The Lick object's predicted motion after August 7 suggests that the day-to-day observing conditions were improving at sunset, but remained poor at sunrise. Of the three occasions on which Campbell failed to recover the object, the best opportunity presented itself unquestionably at sunset on August 8, when the predicted location was about  $5^\circ.3$  from the Sun in position angle of  $\sim 70^\circ$ ; the object would then still have had some 30 hr to go to perihelion. This hypothesis would thus indicate that the object disintegrated more than 30 hr before perihelion.

Finally, we examine the implications of the nearly 100 yr gap between the perihelion times of C/1823 Y1

and the Lick object for the separation velocity at the time of their presumed common parent's fragmentation. With the orbital period of  $2110 \pm 260$  yr (Table A-2), the relation between the statistically averaged separation velocity [governed by Equation (26)],  $\langle V_{\text{sep}} \rangle$  (in  $\text{m s}^{-1}$ ), and the heliocentric distance at separation,  $r_{\text{frg}}$  (in AU), is

$$\langle V_{\text{sep}} \rangle = 6.2^{+1.5}_{-1.1} r_{\text{frg}}^{\frac{1}{2}}. \quad (29)$$

We note that even if the parent comet split right at perihelion, the required separation velocity would be close to  $3 \text{ m s}^{-1}$ , already exceeding the plausible upper limit (Section 2.1.1); and at  $r_{\text{frg}} \simeq 2\text{--}3$  AU it would reach a completely intolerable magnitude of nearly  $10 \text{ m s}^{-1}$ . The only way to avoid this contradiction is to assume that the Lick object survived not one but two (or more) revolutions about the Sun, an option that is rather unattractive. Accordingly, it appears that this scenario fails to satisfy test 6.

We summarize by noting that while the hypothesis of a genetic relationship between the Lick object and comet C/1823 Y1 passes tests 1, 3, and 7, it is, strictly, incompatible with the constraints presented by tests 2, 5, 6, and, to some degree, 4.

### 5.3. Comet C/1847 C1 (Hind)

This is the last of the top three candidates in Table 8 and the most promising one, as argued below. Figure 8 shows that, unlike C/1823 Y1, this comet was approaching the Sun from the high northern declinations; it was

discovered by Hind 52 days before perihelion (Bishop & Hind 1847), when 1.49 AU from the Sun, as a faint telescopic object. The comet brightened steadily and was detected with the naked eye more than three weeks before perihelion (Schmidt 1847a, 1847b; Bond 1847). The discoverer observed the comet telescopically in broad daylight on 1847 March 30, several hours before perihelion, within  $2^\circ$  of the Sun (Hind 1847). As Figure 8 shows, the comet remained close to the Sun during the first few weeks after perihelion. As a result, there were merely *three* astrometric positions available along the receding branch of the orbit, made on April 22–24. Interestingly, around April 12 the projection conditions were favorable for the appearance of an antitail (Sekanina 1976); unfortunately, the comet was then only  $15^\circ$  from the Sun.

Employing a total of 160 astrometric observations made between 1847 February 6 and April 24 and accounting for the perturbations by Mercury through Jupiter, Hornstein (1870) derived an elliptical orbit with a period of  $10\,220 \pm 570$  yr for the osculation epoch at perihelion, implying a true orbital period (i.e., a time interval between 1847 and the previous perihelion) of  $8310 \pm 570$  yr (Marsden et al. 1978). To find out whether we could rely on Hornstein’s computations, we derived two new sets of orbital elements for C/1847 C1 based on the observations made only in London (critical at the beginning of the orbital arc), Berlin (critical at its end), and Vienna. Choice of two different cutoffs for the residuals of rejected observations ( $\pm 10''$  and  $\pm 6''$ ) yielded osculation orbital periods of, respectively,  $8190 \pm 930$  yr and  $14\,100 \pm 2300$  yr. Interestingly, Hornstein (1854a) estimated, in his earlier study based on 145 astrometric observations from the same period of time, that the orbital period was not shorter than 8000 yr and not longer than 14 000 yr. In that early study, Hornstein (1854a, 1854b) established the most probable orbital period of 10 818 yr for the osculation epoch at perihelion; the details of that paper indicate the orbital period’s rms error of  $\pm 1510$  yr. Even though the quality of Hornstein’s final set of orbital elements was classified as only category 2B by Marsden et al. (1978), the numbers for the orbital period appear to be consistent. We conclude that Hornstein’s results are credible enough to indicate that the comet’s true orbital period amounted to about 8000 yr (with an uncertainty of several centuries) and was compatible with the constraints of test 7 in Table 9.

As a companion to C/1847 C1, the Lick object was at the time of Campbell et al.’s observation a little beyond the Sun, so that — unlike in the C/1823 Y1 scenario — its observed brightness was unaffected by forward scattering. On the other hand, the heliocentric distance was now much smaller, which made the object brighter. Accounting for a phase effect (Marcus 2007), we find that the object should have appeared to Campbell et al. about 10.7 mag brighter than was its absolute magnitude with an  $r^{-4}$  law but 13.6 mag brighter with an  $r^{-5}$  law. The estimated apparent magnitude of  $-4.3$  implies for the two laws an absolute magnitude of  $H_0 \approx 6.4$  and 9.3, respectively. We argue that, again, this is a plausible range of absolute magnitude for a companion of an intrinsically bright long-period comet, with the bright end of the range very close to the absolute magnitude of C/1996 Q1, one of the companions to C/1988 A1 investigated in detail in Section 2.2 (Figure 3).

Because the trajectory of the Lick object emulated by C/1847 C1 runs in Figure 8 in a generally southwestern direction on August 6–8, the predicted motion brought the object almost  $2^\circ$  south of the Sun in close proximity of perihelion late on August 8. Viewed from the Lick Observatory, the object was still more than  $2^\circ$  below the horizon at sunrise and already about  $2^\circ$  below the horizon at sunset on August 8, so in this scenario Campbell’s failure to see it was due to unfavorable geometry and not necessarily due to its disintegration. However, by sunrise on August 9 the object swayed already far enough to the west of the Sun that it rose before the Sun, with a net difference of more than  $2^\circ$  in elevation. Hence, Campbell’s failure to see the Lick object at sunrise on August 9, the last occasion on which he was searching for it, cannot be explained by unfavorable geometry. However, the constraint on the object’s disintegration time is relaxed from  $\sim 6$  hr preperihelion to  $\sim 18$  hr *past* perihelion.

A review of the tests in Table 9 shows that they all are satisfied by C/1847 C1. Particularly gratifying is this scenario’s correspondence with the evidence, criteria, and requirements in tests 1–4. The Lick object’s elongation on approach to the Sun is predicted to have dropped below  $50^\circ$  at a heliocentric distance of more than 2 AU and to continue decreasing with time until after perihelion.

The absolute magnitude of C/1847 C1 has not been well determined. For an  $r^{-4}$  law, Vseksvyatsky (1958) gave  $H_{10} = 6.8$ , but the brightness estimates by Schmidt (1847a, 1847b) and by Bond (1847) strongly suggest that between 1.1 AU and 0.5 AU from the Sun the comet brightened less steeply with decreasing heliocentric distance. Correcting a few available magnitude estimates for the phase effect (Marcus 2007) and the geocentric distance, we obtained by least squares  $H_0 = 5.41 \pm 0.13$  and  $n = 2.02 \pm 0.31$ . We note that Holetschek (1913), assuming a brightness variation with an inverse square heliocentric distance derived from a single estimate  $H_0 = 5.7$ , a value that, like Vseksvyatsky’s, was not corrected for a phase effect. Because the phase angle varied between  $55^\circ$  and  $90^\circ$  during the relevant period of time, an average phase correction should make the absolute magnitude brighter by  $\sim 0.4$  to  $\sim 0.7$  mag, depending on the dust content.

Since Figures 1 and 3 show that companion comets have generally steeper light curves than the primaries, the difference between the derived slope for the light curve of C/1847 C1, as the pair’s primary, and the assumed range of slopes ( $n = 4$  to 5) for the light curve of the Lick object, as the companion to C/1847 C1, are not unrealistic. In summary, we find that even if the Lick object was not in terminal outburst when observed by Campbell et al., its absolute magnitude was probably at least 1 mag — and possibly several magnitudes — fainter than the absolute magnitude of C/1847 C1. If the object was in terminal outburst, its quiescent-phase absolute magnitude was yet another magnitude or so fainter.

As the last point of our investigation of C/1847 C1 and the Lick object, as a pair of long-period comets that are likely to have split apart from a common parent near its perihelion some  $8310 \pm 570$  yr before 1847, is their statistically averaged separation velocity  $\langle V_{\text{sep}} \rangle$  (Section 2.2.1), an issue that we focused on in discussing the fragmentation of long-period comets in Sections 2.1 and 2.2. With a temporal gap of 74.4 yr between the

arrivals of C/1847 C1 and the Lick object (Table 9), the separation velocity (in  $\text{m s}^{-1}$ ) is equal to

$$\langle V_{\text{sep}} \rangle = 0.48_{-0.04}^{+0.05} r_{\text{frg}}^{\frac{1}{2}}. \quad (30)$$

If one assumes that the common parent of C/1847 C1 and the Lick object split right at perihelion, the fragments separated at a rate of  $\langle V_{\text{sep}} \rangle = 0.1 \text{ m s}^{-1}$ ; if at a heliocentric distance of  $r_{\text{frg}} \simeq 1 \text{ AU}$ , then  $\langle V_{\text{sep}} \rangle \simeq 0.5 \text{ m s}^{-1}$ ; and if at  $r_{\text{frg}} \simeq 10 \text{ AU}$ ,  $\langle V_{\text{sep}} \rangle \simeq 1.5 \text{ m s}^{-1}$ . This is a typical range of separation velocities as documented by the known split comets (Section 2.1.1; Sekanina 1982).

In spite of the incompleteness of the data on the Lick object, we believe that a fairly compelling case has been presented for its genetic relationship with C/1847 C1, implying that their separation had occurred in the general proximity of the previous perihelion in the 7th millennium BCE. This example demonstrates that the temporal gap between the members of a genetically related pair or group of long-period comets could easily reach many dozens of years and that both the primary and the companion can survive for millennia after the parent comet's splitting. Finally, it is nothing short of remarkable that a premise of the membership in a comet pair can successfully be applied to instances of unidentified interplanetary objects with poorly known motions.

## 6. CONCLUSIONS

The objectives of this investigation were (i) to describe the dynamical properties of the genetically related members of pairs or groups of long-period comets other than the Kreutz system of sungrazers and (ii) by exploiting these findings to determine whether the intriguing and as yet unidentified Lick object, discovered near the Sun at sunset on 1921 August 7, happened to be a member of such a pair and to which long-period comet could it be genetically related. Our conclusions are as follows:

(1) The orbits of genetically related long-period comets are — except for the perihelion time — nearly identical and among objects with dependably determined motions their common origin cannot be disputed.

(2) Up to now, however, only two groups of such genetically related long-period comets have been recognized: a pair of C/1988 F1 (Levy) and C/1988 J1 (Shoemaker-Holt); and a trio of C/1988 A1 (Liller), C/1996 Q1 (Tabur), and C/2015 F3 (SWAN).

(3) On an example of the comet pair of C/1988 F1 and C/1988 J1 we affirm that, of the two mechanisms that evoke a steadily increasing separation of fragments of a split comet with time, an orbital-momentum increment acquired at the parent's breakup is overwhelmingly more consequential for the motions of genetically related long-period comets than is the role of an outgassing-driven differential nongravitational acceleration. Initial velocities (especially the component along the radius vector) that set the fragments apart do not exceed a few  $\text{m s}^{-1}$  and are equivalent to separation velocities of fragments of the known split comets (Sekanina 1982). This result, confirmed by numerical experimentation, makes sense, as the nongravitational effects are utterly trifling far from the Sun and become significant only along a short orbital arc close to perihelion.

(4) The pair of C/1988 F1 and C/1988 J1 differs from the trio C/1988 A1, C/1996 Q1, and C/2001 F3 in that

the pair's parent fragmented only several hundred AU from the Sun after having passed intact through aphelion on its way to the 1987 perihelion, whereas the trio's parent broke up in the general proximity of perihelion during its previous return to the Sun.

(5) We emphasize that the trio's members appear to have differed from one another in their appearance, morphology, and physical behavior, as reflected by their light curves. Only C/1988 A1, presumably the most massive member, showed no sign of imminent disintegration; C/1996 Q1, a dust-poor fragment, lost its nuclear condensation already before perihelion and, appearing as a headless tail, faded progressively afterwards; whereas C/2015 F3 was quite prominent in the SWAN Lyman-alpha atomic-hydrogen images relative to its optical brightness, implying tentatively a high water-to-dust production ratio still to be confirmed by a future research; this most recent fragment survived perihelion seemingly intact, but was fading steeply while receding from the Sun.

(6) The experience with both the pair and the trio of genetically related comets shows the orbits of either group's members so much alike that the near-perihelion motion of one member can readily be emulated by a set of orbital elements of another member, with the perihelion time being the only parameter to be changed; the positions are found to fit within a few tenths of an arcmin for the pair of C/1988 F1 and C/1988 J1, and to better than  $10'$  for the trio of C/1988 A1, C/1996 Q1, and C/2015 F3.

(7) This remarkable match suggests that a meaningful search for a member of an as yet unrecognized pair or group of genetically related comets can be carried out by scouting around for a comet whose motion offers this orbital correspondence, even when the astrometric data for the examined object are not sufficient to compute a set of orbital elements in a standard fashion.

(8) This idea is tested on a celebrated bright object of 1921 August 7, whose discovery near the Sun was made with the naked eye at sunset by a group of people, including two prominent astronomers, from the premises of the Lick Observatory. However, the Lick object was searched for unsuccessfully at sunrise on August 8 and 9 and at sunset on August 8. It was definitely not a Kreutz sungrazer and its identity has remained shrouded in mystery to this day.

(9) The search for a comet that could be genetically related to the Lick object was based primarily on the object's estimated position at sunset on August 7, published by Campbell and believed to be accurate to about  $\pm 10'$ ; this was the primary test.

(10) Two independent, though less accurate observations made on 1921 August 6–7 as well as additional constraints — such as the requirement of an unfavorably oriented approach trajectory in the sky (at persistently small elongations) to explain the failure to detect the Lick object earlier; or a preference for the related comet to have arrived at perihelion prior to the Lick object — were employed to develop additional tests.

(11) Brightness arguments strongly suggested that the Lick object's close proximity to the Sun was not merely an effect of projection, but that the actual distance between the object and the Sun was indeed very small when observed at the Lick Observatory.

(12) Accordingly, among comets with perihelion distances not exceeding 0.25 AU (a conservative upper limit) we searched for a comet that could be genetically related to the Lick object; we found three candidates that satisfied the primary constraint based on the Campbell report: C/1689 X1, C/1823 Y1, and C/1847 C1 (Hind).

(13) Close examination of the past work on the motion of C/1689 X1 showed that it is essentially indeterminate; an apparent match of one of the comet’s many different orbits to the Lick object must be fortuitous. The orbit of C/1823 Y1 fails to satisfy up to four of the seven tests, including a limit on the separation velocity at breakup.

(14) Only the orbit of C/1847 C1 is fully compatible with all applied tests, suggesting that if the Lick object is a fragment of another comet, C/1847 C1 is by far the most likely primary fragment of the same parent body.

(15) In terms of the intrinsic brightness, the Lick object was probably at least 1 mag fainter than C/1847 C1; the temporal gap of 74.4 yr between the perihelion times suggests that they separated — with a relative velocity not exceeding  $\sim 1.5 \text{ m s}^{-1}$  and probably lower than  $1 \text{ m s}^{-1}$  — from their parent during its breakup in a general proximity of perihelion at its return to the Sun in the course of the 7th millennium BCE.

(16) Campbell’s failure to recover the object at sunrise on August 9 could be due to its disintegration near perihelion, an event also experienced by C/1996 Q1.

(17) The example of the 1921 Lick object shows that members of a genetically related pair or group of long-period comets could be separated from one another by many dozens of years and that both the primary and the companion can survive for thousands of years after their parent’s splitting. The proposed method of search merits applications to other instances of unidentified objects with poorly known motions.

(18) As part of our examination of the true orbital periods of the promising candidate comets, we recomputed the orbital elements of C/1823 Y1 and established that its true orbital period was  $\sim 2100$  yr and that the “definitive” parabolic orbit by Hnatek (1912) in the Marsden-Williams (2008) catalog is misleading.

This research was carried out in part at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

## APPENDIX

### NEW ORBIT DETERMINATION FOR C/1823 Y1

We remarked in Section 5.2 that this comet’s orbit determination by Hnatek (1912), although the most comprehensive on record, suffered from a defect in that he presented a parabola as his “definitive” solution without submitting any compelling evidence to demonstrate that the orbital period was indeterminate.

In the process of refining the orbit, Hnatek deduced separately the sets of elements for both the “most probable parabola” and the “most probable ellipse.” The eccentricity of this elliptical orbit was listed by the autor as 0.9995048. In addition, Hnatek also provided the sums of squares of residuals from his 8 normal places not only for the most probable parabola and ellipse, but also for

four additional forced eccentricities between 0.9994 and 1.0002. Hnatek’s paper shows that the sum of squares of residuals for the most probable ellipse was 0.8 the sum of squares of residuals for the most probable parabola, which by itself suggests that the two solutions were by no means equivalent. Yet, immediately following the table of the residual sums, Hnatek addressed this issue very differently, saying that “as seen from [the table], one can vary the orbital period to rather widely stretched limits without running into conflict with the observations and can thereby conclude that a parabola can ultimately be adopted as a definitive shape of the orbit.” Then he proceeded to optimize the parabolic solution after discarding the last normal place in both right ascension and declination because of the residuals that reached  $10''$  to  $20''$ . This curtailed the orbital arc covered by the observations by more than three weeks, from 92 days down to 68 days.

From the entries in the table of the sums of squares of residuals nearest Hnatek’s most probable ellipse we found that the mean error of the eccentricity was  $\pm 0.000120$ , whereas the deviation of the eccentricity from unity for this solution was 0.000495, more than a factor of 4 higher. This result suggests that the choice of the parabolic approximation was unjustified. Since the discarded normal place was based on 8 observations between 1824 March 17 and 31, while the previous two normal places contained only 4 observations each, from February 29 through March 3 and from March 4 through March 7, respectively, it probably was a mistake to discard the last one, with the greatest weight of the three. We feel that this phase of Hnatek’s work on the orbit of C/1823 Y1 precluded him from capitalizing on the great potential that his project had offered.

Under the circumstances, our decision to go ahead and undertake the task of redetermining the orbit of comet C/1823 Y1 from scratch was motivated by two issues. One, as this comet was potentially genetically related to the Lick object, the requirement that its true orbital period be confined to particular limits was one of the critical conditions that had to be satisfied. Thus, the orbit’s ellipticity was urgently needed. And, two, it was desirable to update the astrometric positions of the comparison stars used by Hnatek with the more accurate ones from the *Hipparcos* and *Tycho-2* catalogs.<sup>16</sup>

To collect the data for our computations, whose results are summarized in Tables A-1 and A-2, we employed the comet’s observed offsets, in right ascension and declination, from the comparison stars identified for a majority of the astrometric observations by Hnatek (1912).<sup>17</sup> When, at a given time, a comparison of the comet’s position with a star’s position was made only in right ascension or only in declination, the offset in the other coordinate was determined by interpolating the neighboring offsets only when measured from the same comparison star and when involving intervals not exceeding  $\sim 1$  hr; no offsets were ever extrapolated.

<sup>16</sup> The search facilities are available at the following websites: <http://www.rssd.esa.int/index.php?project=HIPPARCOS&page=hipsearch> for the *Hipparcos* (and the original *Tycho*) catalog and <http://vizier.u-strasbg.fr/viz-bin/VizieR-3?-source=I/259/tyc2&-out.add=> for the *Tycho-2* catalog.

<sup>17</sup> A sizable minority of the observations was available only as the comet’s apparent positions with no comparison stars specified; these observations were ignored.

**Table A-1**  
RESIDUALS FROM THE ORBITAL SOLUTION FOR C/1823 Y1 BASED ON 102 ASTROMETRIC OBSERVATIONS  
BETWEEN 1824 JANUARY 2 AND MARCH 31 (EQUINOX J2000).

Time of observation 1824 (UT)	Residual in		Observ- atory	Time of observation 1824 (UT)	Residual in		Observ- atory	Time of observation 1824 (UT)	Residual in		Observ- atory
	R.A.	Dec.			R.A.	Dec.			R.A.	Dec.	
Jan. 2.23975	-8''2	-5''3	Paris	Jan. 15.22277	-1''7	+6''1	Marseilles	Feb. 7.24658	+4''2	-5''1	Marseilles
2.28741	+5.1	-2.4	Greenwich	15.22806	-0.7	-2.9	"	7.88922	-6.3	-4.9	Nikolayev
2.28883	+9.1	+6.3	"	15.22888	-0.2	-4.3	"	10.74380	+0.1	-8.1	Vienna
4.25867	-3.4	+1.1	Mannheim	15.23419	+2.3	+6.7	"	12.22860	-3.1	-0.8	Marseilles
5.19242	-2.1	+2.3	"	15.23506	+2.0	+8.4	"	17.76379	-9.8	+0.7	Vienna
6.19679	-2.0	-4.8	"	15.24029	+0.2	-2.4	"	18.78077	+10.5	-1.6	Mannheim
6.21341	+6.2	-6.2	Vienna	16.03822	+0.4	-4.6	Nikolayev	22.90792	+6.2	+2.6	"
7.16077	-3.3	+8.3	"	16.04074	-8.3	+4.4	"	24.80239	-0.8	-3.1	Nikolayev
7.16077	-1.9	+4.2	"	18.08508	-5.1	-3.8	"	24.81977	+1.5	-1.1	"
7.19604	+1.0	-0.2	Marseilles	19.15802	-1.6	-8.1	Marseilles	24.86462	+6.4	-6.6	"
7.20727	-4.8	+7.6	"	19.18314	+3.1	-6.7	"	27.92409	-5.9	-3.4	"
7.20815	-4.3	+8.2	"	19.18422	+3.7	-5.9	"	27.93371	+5.5	-1.8	Mannheim
7.21940	+5.4	-7.0	"	19.19462	+9.6	+4.1	"	27.94977	+1.8	+8.9	Nikolayev
7.21940	+2.9	-8.9	"	24.99474	+6.9	-0.5	"	27.95905	+5.7	+2.6	"
9.20090	+2.3	-6.4	"	27.04860	+11.6	+1.2	Nikolayev	29.81069	-0.7	-2.8	"
9.22572	+7.4	-1.5	"	30.85869	+0.2	+10.4	Marseilles	29.82372	-4.4	-3.8	"
9.25445	+0.4	-4.4	"	30.85869	+0.6	-1.7	"	29.83197	-2.3	-7.7	"
9.25725	-3.0	+1.2	"	30.85869	-0.9	+6.8	"	29.84108	-0.4	-0.6	"
9.25805	-2.0	+2.8	"	30.86334	+2.7	+4.1	"	Mar. 2.93493	-4.0	-6.7	Dorpat <sup>a</sup>
9.26293	+4.0	-2.7	"	30.86334	+2.8	-1.0	"	4.83756	-2.9	-3.4	Nikolayev
10.24861	-7.7	+2.8	Mannheim	30.86334	-0.8	+2.6	"	4.84897	+3.1	+8.4	"
12.22219	+3.7	-6.4	Marseilles	30.86712	+0.4	-1.0	"	4.85972	+10.4	-0.2	"
12.24189	-4.5	-6.8	"	30.86712	+2.0	-0.5	"	4.86952	+6.0	+6.8	"
12.24543	+1.7	-6.8	"	30.86712	-0.8	-0.8	"	4.94224	+0.9	+2.4	Mannheim
12.24619	+3.9	-6.7	"	30.87187	-2.6	-4.1	"	6.86113	-7.7	-3.3	Nikolayev
13.20964	+5.3	-1.3	Mannheim	30.87187	+0.9	+2.0	"	6.95187	-5.9	+6.2	"
13.22120	-7.4	+6.3	Marseilles	30.87187	-0.8	+5.6	"	6.97363	-7.7	-3.0	"
13.23014	-8.0	+6.3	"	Feb. 1.85089	+1.4	-7.0	"	18.77981	-2.8	+1.9	"
14.19640	+0.8	+8.3	"	1.87081	+4.1	-10.3	"	18.78533	+2.2	+1.8	"
14.20295	-3.3	+6.7	"	2.75793	+3.2	+2.1	Mannheim	20.79306	-1.2	-4.9	"
14.20384	-2.6	+6.5	"	4.75398	-8.3	-5.3	Vienna	20.79955	-7.9	+6.8	"
14.21009	+1.4	+4.5	"	4.75398	-6.6	-3.1	"	21.78829	+5.6	+8.4	"
14.21106	+0.4	+4.2	"	4.80650	+8.1	+8.7	Marseilles	22.78238	+2.0	+1.5	"
14.21903	-7.0	+2.8	"	4.82240	-3.7	-3.0	"	31.89001	+6.4	+4.6	"

<sup>a</sup> Nowadays: Tartu, Estonia.

**Table A-2**  
NEW ORBITAL ELEMENTS FOR COMET C/1823 Y1 AND COMPARISON WITH HNATEK'S ORBIT  
(EQUINOX J2000.0).

Quantity/Orbital element	New orbit	Differences Hnatek's minus new orbit
Osculation epoch (TT)	1823 Dec. 11.0	(1824 Feb. 15.0)
Time of perihelion passage $t_\pi$ (TT)	1823 Dec. 9.94585 $\pm$ 0.00063	-0.0116
Argument of perihelion $\omega$	28° 2778 $\pm$ 0° 0160	+0° 2073
Longitude of ascending node $\Omega$	305° 5647 $\pm$ 0° 0046	-0° 0589
Orbit inclination $i$	103° 6810 $\pm$ 0° 0110	+0° 1374
Perihelion distance $q$	0.225238 $\pm$ 0.00011	+0.001492
Orbital eccentricity $e$	0.998698 $\pm$ 0.00010	+0.001302
Orbital period (yr) $\left\{ \begin{array}{l} \text{osculation} \\ \text{original}^a \end{array} \right.$	2280 $\pm$ 260	.....
	2110	.....
Orbital arc covered by observations	1824 Jan. 2–1824 Mar. 31	.....
Number of observations employed	102	.....
Root-mean-square residual	$\pm 5''00$	.....
Orbit-quality code <sup>b</sup>	2B	.....

<sup>a</sup> With respect to the barycenter of the Solar System.

<sup>b</sup> Following the classification system introduced by Marsden et al. (1978).



Once the positions of the relevant comparison stars were identified in the *Hipparcos* catalog (in more than 90% of cases) or the *Tycho-2* catalog, a total of 388 positions of the comet were re-reduced and incorporated into a differential least-squares optimization procedure to compute an initial set of orbital elements by employing the *EXORB7* code.

The presence of a dozen observed positions with enormous residuals, exceeding  $\pm 10'$ , suggested that several comparison stars were apparently misidentified by the observers (or by Hnatek). After removing these bad data, we ended up with a total of 376 observations, which served as an input to another orbit iteration. Although we still found more than a dozen observations whose residuals (in at least one coordinate) exceeded  $\pm 200''$ , the osculation orbital period already came out to be close to 2000 yr, with an uncertainty of about  $\pm 50\%$ . After the most inferior data were removed, the orbit was iterated again with 360 observations left in the solution. Next, 32 more positional data were removed with the residuals exceeding  $\pm 100''$  and a fourth iteration was carried out. We then continued with a fifth iteration based on 280 observations with the residuals not exceeding  $\pm 50''$ ; with a sixth iteration based on 178 observations with the residuals not exceeding  $\pm 25''$ ; and with a seventh iteration based on 102 observations with the residuals not exceeding  $\pm 12''$ ; the rms residual of this solution was  $\pm 5''.0$ . We tested an eighth iteration based on 93 observations with the residuals not exceeding  $\pm 10''$ , but even though the rms residual dropped to  $\pm 4''.6$ , the errors of the orbital elements remained essentially unchanged from the solution derived in the seventh iteration, which thus was the final result of the computations.

The residuals from the 102 employed observations are listed in Table A-1, while the elements for the standard 40-day osculation epoch nearest perihelion are presented in Table A-2. This table also shows the differences between our and Hnatek's results (at the osculation epoch of Hnatek's choice, 1824 February 15). The differences are more than a factor of 10 greater than the errors of our set of elements. The original reciprocal semimajor axis is equal to  $(1/a)_{\text{orig}} = +0.006077 \pm 0.000471 \text{ (AU)}^{-1}$  and the true orbital period to  $P_{\text{orig}} = 2110 \pm 260 \text{ yr}$ . The comet's orbit is decidedly elliptical and the true orbital period, derived with a 12% uncertainty and compatible with test 7, is much shorter than the orbital period of Hnatek's most probable ellipse. The comet definitely did not arrive from the Oort Cloud.

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